DESIGNING ROBUST QUANTUM REFRIGERATORS IN DISORDERED SPIN MODEL

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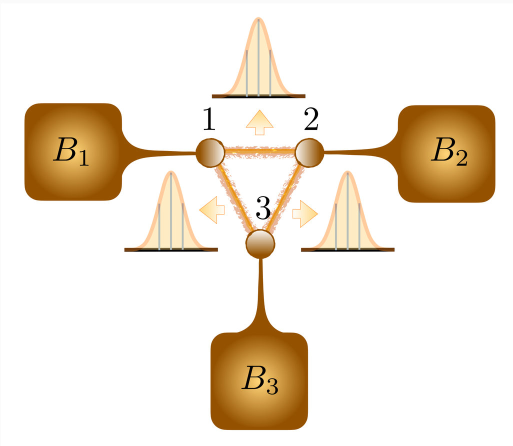
Abstract

We explore a small quantum refrigerator in which the working substance is made of paradigmatic nearest-neighbor quantum spin models, the XYZ and the XY model with Dzyaloshinskii-Moriya interactions, consisting of two and three spins, each of which is in contact with a bosonic bath. We identify a specific range of interaction strengths which can be tuned appropriately to ensure a cooling of the selected spin in terms of its local temperature in the weak- coupling limit. Moreover, we report that in this domain, when one of the interaction strengths is disordered, the performance of the thermal machine operating as a refrigerator remains *almost* unchanged instead of degradation, thereby establishing the flexibility of this device. However, to obtain a significant amount of cooling via ordered as well as disordered spin models, we observe that one has to go beyond the weak-coupling limit and compute the figures of merits by using global master equations.

Design of Refrigerator



$$H_F = \sum_{i=1}^{N} h_i \sigma_z^i, \quad (1)$$

$$H_{xy} = \sum_{i=1}^{N} J_{i,i+1}^{xy} \left[(1+\gamma)\sigma_x^i\sigma_x^{i+1} + (1-\gamma)\sigma_y^i\sigma_y^{i+1}\right] \quad (2)$$

$$H_z = \sum_{i=1}^{N} J_{i,i+1}^{z}\sigma_z^i\sigma_z^{i+1}, \quad (3)$$

$$H_{dm} = \sum_{i=1}^{N} J_{i,i+1}^{dm}\left(\sigma_x^i\sigma_y^{i+1} - \sigma_y^i\sigma_x^{i+1}\right). \quad (4)$$

- γ is the xy anisotropy parameter, h_i is the strength of the local magnetic field acting on the spin i
- The initial state of the system is given by $\rho_s^0 = \bigotimes_{i=1}^N \rho_i^0$, where $\rho_i^0 = \exp\left(-\beta_i^0 h_i \sigma_z^i\right)/\text{Tr}\left[\exp\left(-\beta_i^0 h_i\sigma_z^i\right)\right]$, with $\beta_i^0 = (k_B T_i^0)^{-1}$, k_B is the Boltzmann constant.

Master Equations

- Local Master Equation

$$\mathcal{D}_i(\rho) = \Gamma_i\Big[(n_\omega^i+1)\big(\sigma_i^-\rho\sigma_i^+ - \frac{1}{2}\{\sigma_i^+\sigma_i^-,\rho\}\big) + n_\omega^i\big(\sigma_i^+\rho\sigma_i^- - \frac{1}{2}\{\sigma_i^-\sigma_i^+,\rho\}\big)\Big], \quad (5)$$

- Global Master Equation

$$\mathcal{D}_i(\rho) = \sum_{\omega>0}\gamma_i^\omega\Big[\big(A_\omega^i\rho A_\omega^{i\dagger} - \frac{1}{2}\{A_\omega^{i\dagger}A_\omega^i,\rho\}\big) + \big(A_\omega^{i\dagger}\rho A_\omega^i - \frac{1}{2}\{A_\omega^i A_\omega^{i\dagger},\rho\}\big)\Big], \quad (6)$$

where the operator A_ω^i is given by

$$e^{iH_st}(\sigma_i^+ + \sigma_i^-)e^{-iH_st} = 2\sum_\omega A_\omega^i e^{-i\omega t} \quad (7)$$

Local Refrigeration

- Local Temperature

$$T_i(t) = \frac{2h_i}{\ln\left[\tau_i(t)^{-1} - 1\right]} \quad (8)$$

- A local steady-state cooling of the spin i is achieved if

$$T_i^s = T_i(t\to\infty) < T_i^0 \quad (9)$$

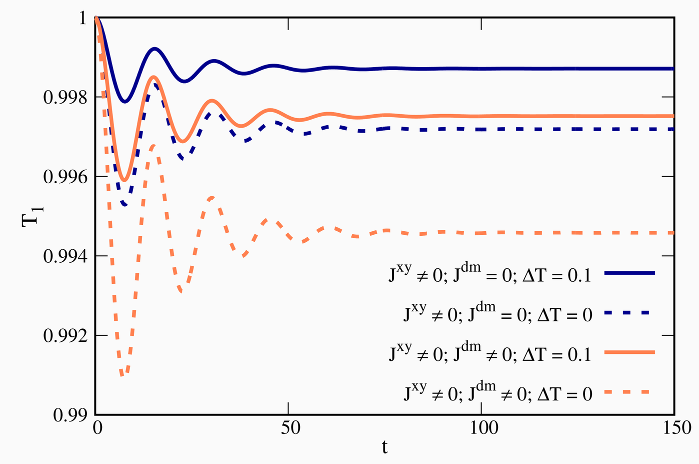
where T_i^0 denotes initial temperature of the spin i

Disorder

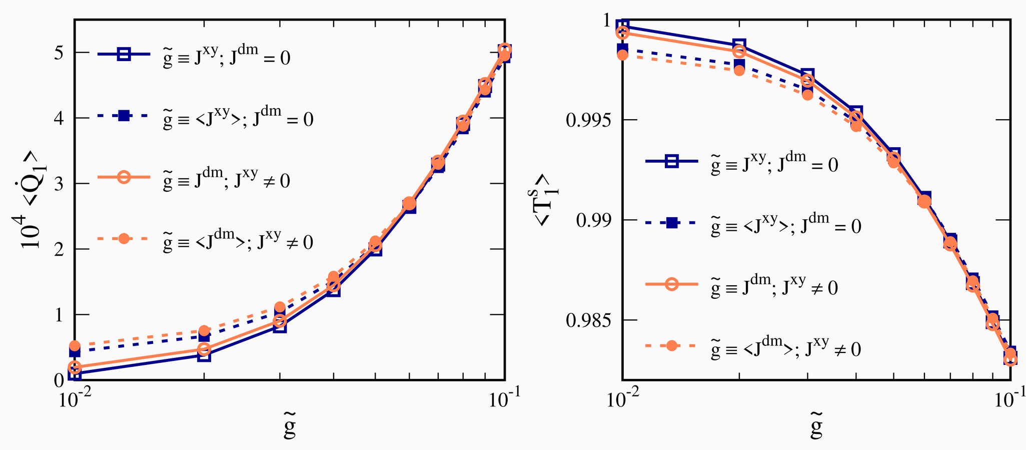
$$\langle\mathcal{Q}(\langle g\rangle,\sigma_g)\rangle = \int \mathcal{P}(g)\mathcal{Q}(g)d(g), \quad (10)$$

where g is the parameter values, which are chosen from a Gaussian distribution ($\mathcal{P}(g)$) of mean $\langle g\rangle$ and standard deviation σ_g quantifying the strength of the disorder.

Quantum refrigerator: Order vs. disorder

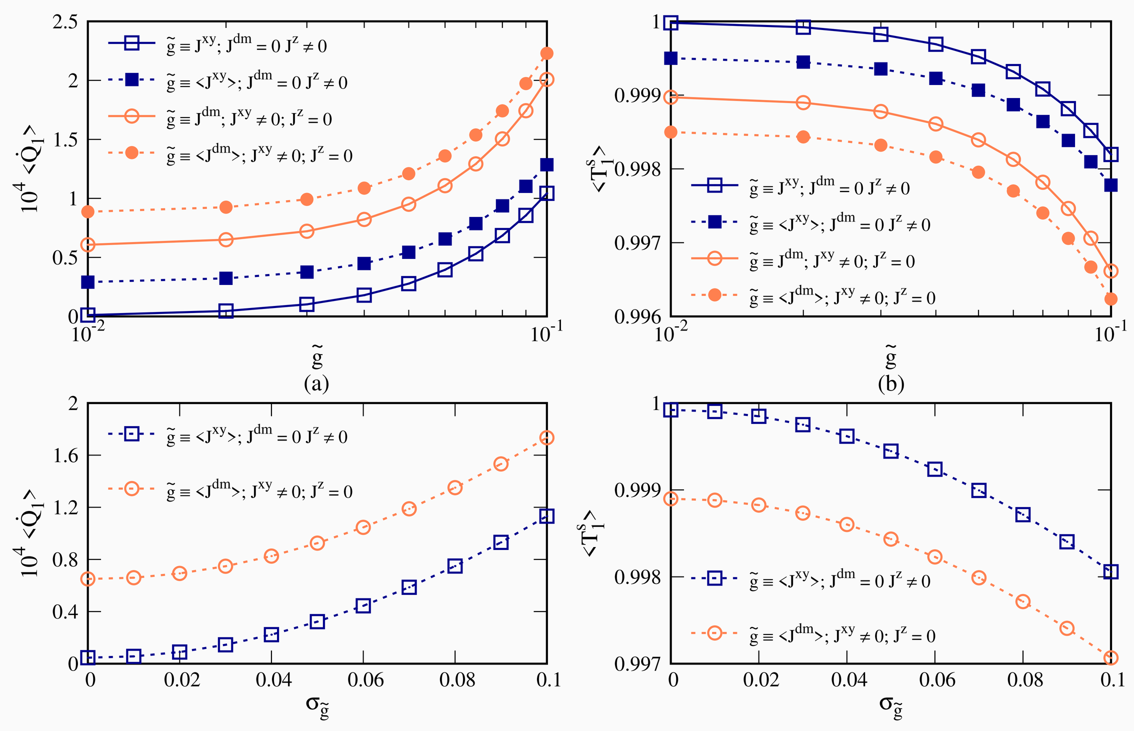


- A non-zero XY interaction strength, J^{xy}, results in an evolution of the system, leading to a local cooling of spin 1, irrespective of the value of J^z.
- More importantly, we report that vanishing $\Delta T = T_2^0 - T_1^0$ proves to be advantageous with respect to cooling than that of a non-vanishing ΔT if we suitably adjust the parameters of H_s and the spin-bath interaction strength (comparing solid ($T_2^0 = 1.1, T_1^0 = 1$) and dashed lines ($T_2^0 = T_1^0 = 1$)).



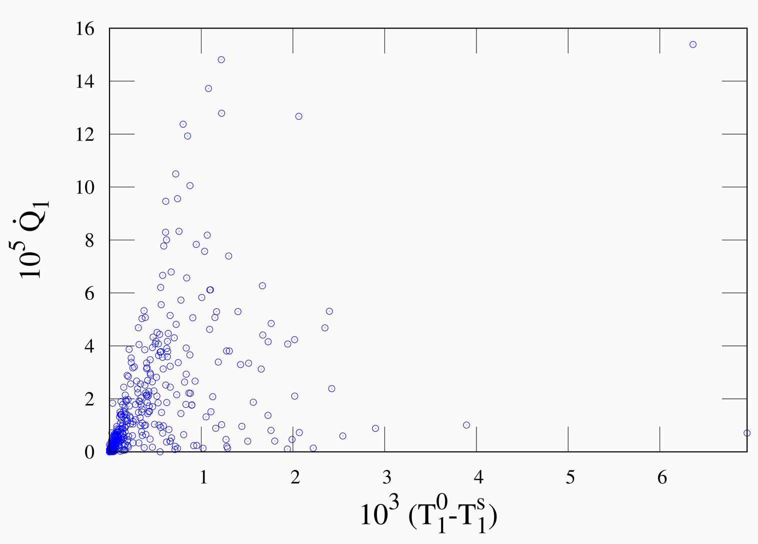
(a) (b)

The heat current (the steady-state temperature) remains almost constant when the strength of the spin-exchange interaction is $\leq 10^{-2}$, and increases with an increase in the value of J^{xy} within the weak-coupling limit ($\leq 10^{-1}$), irrespective of the presence of the interactions in the z-plane, i.e., independent of the values of J^z.



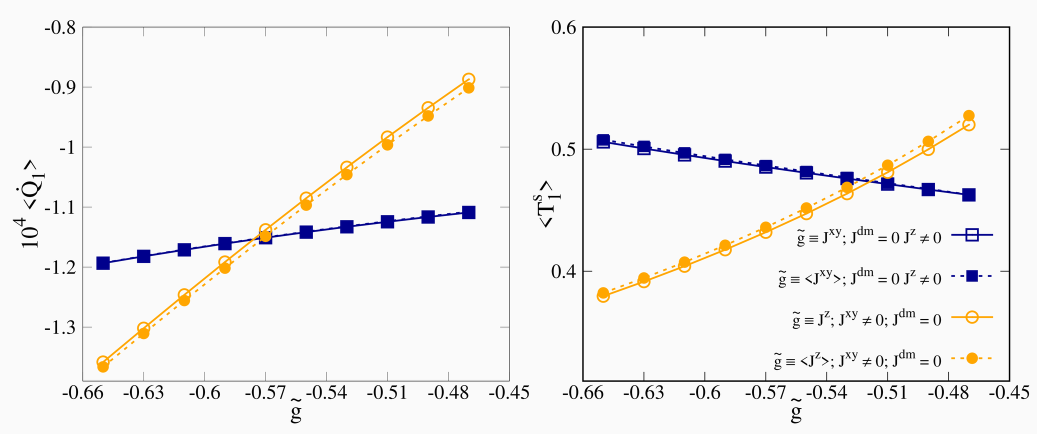
(a) (b) (c) (d)

- We compare the performance of refrigerator designed via ordered and disordered three spin model. It shows robustness of refrigeration in presence of disorder in the system ((a)-(b)).
- We systematically increase the value of the disorder-strength up to 10^{-1}, the average value of the heat current of the first spin attains a more positive value, while the steady-state temperature becomes lower ((c)-(d)) than that of the model with low disorder-strength. It clearly exhibits an advantage to attain a lower steady-state temperature of the refrigerated spin in the presence of disorder.



The values of the local magnetic fields, $\{h_1, h_2, h_3\}$, corresponding to the individual spins are chosen uniformly from $[1.1, 5]$ while the values of the spin-bath interaction parameters $\{\Gamma_1, \Gamma_2, \Gamma_3\}$ as well as the spin-exchange interaction strengths $\{J^{xy}, J^z\}$ are chosen from a uniform distribution of range $[0, 10^{-1}]$. Here $T_1^0 = 1$, $T_2^0 = 2$ and $T_3^0 = 3$. Among 10^4 choices of parameters, only 4.11% points are displayed for which local temperature of the first spin is lower than unity which indicate that there is no monotonic relation between them.

Beyond weak-coupling limit



(a) (b)

- We find that the steady-state temperature and the corresponding quenched averaged temperature of the first spin can substantially be decreased in the strong-coupling domain compared to that obtained in the weak-coupling limit.
- While the robustness of local cooling in the disordered refrigerator is a common feature in both local and global master equations, an interesting difference between these two situations emerge.

References

1. N. Linden, S. Popescu, and P. Skrzypczyk, Phys. Rev. Lett. **105**, 130401 (2010).
2. T. K. Konar, S. Ghosh, A. K. Pal and A. Sen (De), Phys. Rev. A **105**, 022214 (2022).