Andreev-Coulomb heat engines

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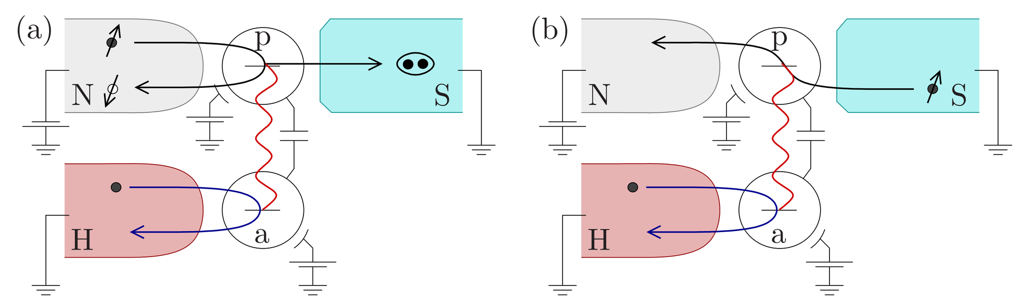
Abstract Three-terminal arrangements of quantum dots can work as energy harvesters. Their capacitive coupling allow for the separation of charge and heat currents, as well as for the isolation of the heat source. In normal systems, the heat to charge conversion depends on tiny energy-dependent asymmetries of the device [1, 2]. We show that larger currents are expected in hybrid systems, where a the presence of a superconductor induces the necessary symmetry breaking [3]. This can be due either to the hybridization of even-parity states (Cooper-pair regime), or to the gap (quasiparticle regime). In the latest case, the system also works as a refrigerator.



Coulomb coupled quantum dots

We consider two capacitively coupled quantum dots. The **active** ($\alpha{=}a$) dot is tunnel-coupled to a hot terminal (H) at temperature T_H. The **passive** ($\alpha{=}p$) dot is coupled to a normal (N) and a superconducting (S) terminal. The passive system is isothermal, with its leads being at temperature T and chemical potential $\mu_N = \mu_S - eV$.
Our model Hamiltonian of the system can be written as

$$\hat{H} = \hat{H}_{\rm dqd} + \hat{H}_{\rm leads} + \hat{H}_t,$$

where

$$\hat{H}_{\rm dqd} = \sum_\alpha \varepsilon_\alpha \hat{n}_\alpha + U_p \hat{n}_{p,\uparrow}\hat{n}_{p,\downarrow} + U_{ap}\hat{n}_a\hat{n}_p,$$

is the double quantum dot Hamiltonian with energy levels ε_α, intra- (U_p) and interdot (U_{ap}) charging energies and number operators $\hat{n}_p = \Sigma_\sigma \hat{n}_{p,\sigma} = \Sigma_\sigma \hat{d}^\dagger_{p,\sigma}\hat{d}_{p,\sigma}$ with spin $\sigma = \{\uparrow,\downarrow\}$ and $\hat{n}_a = \hat{d}^\dagger_a \hat{d}_a$, where $\hat{d}_{p,\sigma}$ and $\hat{d}_a$ are the electron annihilation operators in passive and active dots (for simplicity, the active dot is described in terms of spinless electrons). $\hat{H}_{\rm leads}$ and $\hat{H}_t$ are the leads and the lead-dot tunneling Hamiltonians, respectively:

$$\hat{H}_{\rm leads} = \sum_k \varepsilon_{k,H}\hat{c}^\dagger_{k,H}\hat{c}_{k,H} + \sum_{k,,l=\{N,S\},\sigma} \varepsilon_{k,l}\hat{c}^\dagger_{k,l,\sigma}\hat{c}_{k,l,\sigma} + \sum_k \Delta\left(\hat{c}^\dagger_{k,S,\uparrow}\hat{c}^\dagger_{k,S,\downarrow} + {\rm h.c.}\right)$$

$$\hat{H}_t = \sum_k \left(t_H \hat{d}^\dagger_a \hat{c}_{k,H} + {\rm h.c.}\right) + \sum_{k,\sigma}\left(t_N \hat{d}^\dagger_{p,\sigma}\hat{c}_{k,N,\sigma} + t_S \hat{d}^\dagger_{p,\sigma}\hat{c}_{k,S,\sigma} + {\rm h.c.}\right)$$

Tunnel hybridization: $\Gamma_\beta = 2\pi|t_\beta|^2\rho_0^\beta$.

Methods:

We use non-equilibrium Green's functions techniques for the numerical calculations (expanding the interaction self-energies up to 2nd. order).

In the appropriate limiting cases, we compare these results with master equation approaches:

- $\Delta \gg U_{ap},\ U_p,\ k_{\rm B}T$ (pair transport)
- $k_{\rm B}T \sim U_{ap},\ U_p,\ k_{\rm B}T$ (quasiparticle transport)

NEGF:

Let us skip all details, and just write the currents ($\alpha{=}a,p$, $l{=}N,S,H$):

$$I_l = \frac{e}{2h}\int d\omega \mathcal{I}_l(\omega) \qquad J_l = \frac{e}{2h}\int d\omega(\omega-\mu_l)\mathcal{I}_l(\omega)$$

$$\mathcal{I}_l(\omega) = {\rm Tr}\Big[\hat{\sigma}_z(\boldsymbol{G}^R_\alpha(\omega)\boldsymbol{\Sigma}^<_{\alpha,l}(\omega) + \boldsymbol{G}^<_\alpha(\omega)\boldsymbol{\Sigma}^A_{\alpha,l}(\omega) - \boldsymbol{\Sigma}^R_{\alpha,l}(\omega)\boldsymbol{G}^<_\alpha(\omega) - \boldsymbol{\Sigma}^<_{\alpha,l}(\omega)\boldsymbol{G}^A_\alpha(\omega)\Big].$$

with the corresponding quantum dot self-energies, $\Sigma^{R/<}_{\alpha,l}$, and Keldysh-Green functions, $G^{R/<}_\alpha$. See Ref. [3] for details.

Rate equations:

(Valid for $k_{\rm B}T \gg \Gamma_\beta$)
Transition rates for $|\kappa\rangle\to|\lambda\rangle$ from terminal l into quantum dot α:
$\Gamma^{\alpha l}_{\kappa\lambda}$ (for electrons), $\gamma^{\alpha l}_{\kappa\lambda}$ (for holes)

$$0 = \sum_{\alpha,\beta,\kappa}\left(\Gamma^{\alpha\beta}_{\kappa\lambda}P(\lambda) - \gamma^{\alpha\beta}_{\lambda\kappa}P(\kappa)\right)$$

$P(\lambda)$ is the occupation probability for state $|\lambda\rangle$. We get the currents:

$$I_p = \sum_{\lambda,\kappa}\left(\Gamma^{pN}_{\lambda\kappa} - \gamma^{pN}_{\lambda\kappa}\right)P(\kappa) \qquad J_l = \sum_{\lambda,\kappa}(E_\lambda - E_\kappa - \mu_l)\left(\Gamma^{\alpha l}_{\lambda\kappa} - \gamma^{\alpha l}_{\lambda\kappa}\right)P(\kappa).$$

Power: $P = (\mu_N - \mu_S)I_p > 0$ when generated
Heat: $J_l > 0$ when out of terminal l

Conclusions

- Current under the competition of Cooper pairs and quasiparticles
- The crossover between the two regimes is marked by a sign change
- Andreev-Coulomb current due to asymmetry of the even-parity hybridization [3]
- Efficient energy harvesting [1, 2] and hybrid operations [4]

References

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Andreev-Coulomb regime

Consider the infinite-gap, weak-coupling limit.
The superconductor is described by an effective term: $\hat{H}_{\rm pairing} = \Gamma_S\left(\hat{d}^\dagger_{p,\uparrow}\hat{d}^\dagger_{p,\downarrow} + {\rm h.c.}\right)$

- Quantum dot states (n=0,1: occupation of the active dot):

  Even: $|\pm,n\rangle = \mathcal{N}^{-1}_{\pm,n}(A_{\pm,n}|0,n\rangle - \Gamma_S|2,n\rangle)$, **Odd:** $|\sigma,n\rangle$

  $$A_{\pm,n} = \tilde{\varepsilon}_n \pm \sqrt{\tilde{\varepsilon}_n^2 + \Gamma_S^2}, \quad \tilde{\varepsilon}_n = \varepsilon_p + nU_{ap} + U_p/2$$

- Transition rates:

  $$\Gamma^{\alpha l}_{\lambda\kappa} = \mathcal{G}^{\alpha l}_{\lambda\kappa} f_l(E_\lambda - E_\kappa)\ \text{(an electron)} \qquad \gamma^{\alpha l}_{\lambda\kappa} = \mathcal{J}^{\alpha l}_{\lambda\kappa}[1 - f_l(E_\kappa - E_\lambda)]\ \text{(a hole)}$$

  $$\mathcal{G}^{\alpha l}_{\lambda\kappa} = \Gamma_l|\langle\lambda|\hat{\delta}^\dagger_\alpha|\kappa\rangle|^2 \qquad \mathcal{J}^{\alpha l}_{\lambda\kappa} = \Gamma_l|\langle\lambda|\hat{\delta}_\alpha|\kappa\rangle|^2 \qquad (\hat{\delta}_a \equiv \hat{d}_a,\quad \hat{\delta}_p \equiv \sum_\sigma \hat{d}_{p,\sigma})$$

  $f_l(E) = \{1 + \exp[(E - \mu_l)/k_{\rm B}T_l]\}^{-1}$ is the Fermi-Dirac function.

☞ Fluctuations $|+,n\rangle \leftrightarrow |-,n\pm1\rangle$ change the shape of the wavefunction.

☞ They can be due to electron or hole tunneling.

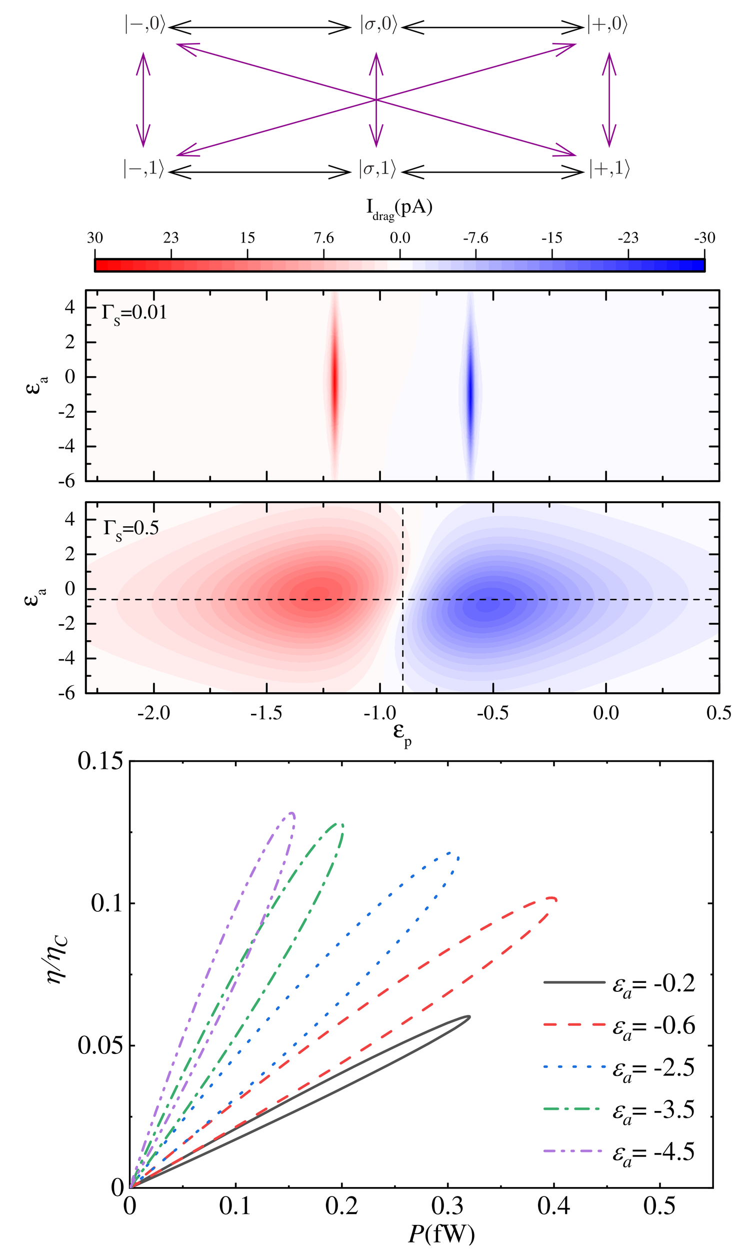
$$r_{+n} = \frac{\mathcal{G}^{pN}_{+n,\sigma n}}{\mathcal{J}^{pN}_{+n,\sigma n}} = \frac{\Gamma_S^2}{A^2_{+n}} \qquad r_{-n} = \frac{\mathcal{G}^{pN}_{\sigma n,-n}}{\mathcal{J}^{pN}_{\sigma n,-n}} = \frac{A^2_{-n}}{\Gamma_S^2}$$

- Sequences of the form:

  $$|\sigma,n\rangle \to |+,n\rangle \leftrightarrow |-,n'\rangle \to |\sigma,n'\rangle \leftrightarrow |\sigma,n\rangle,$$

  break electron-hole symmetry dynamically if $r_{+n}r_{-n'} > 1$.

☞ The sign of the current is tuned with gate voltages.



☞ The symmetry of the wavefunction determines the sign of the current!

Quasiparticle regime

Neglect pairing and doubly occupied states.

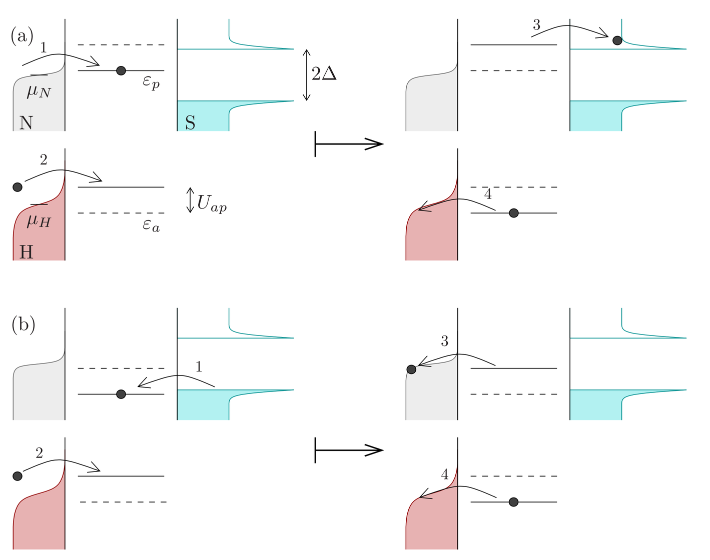
- Passive dot (charge) states: $|0,n\rangle$, $|1,n\rangle$
- Transition rates:

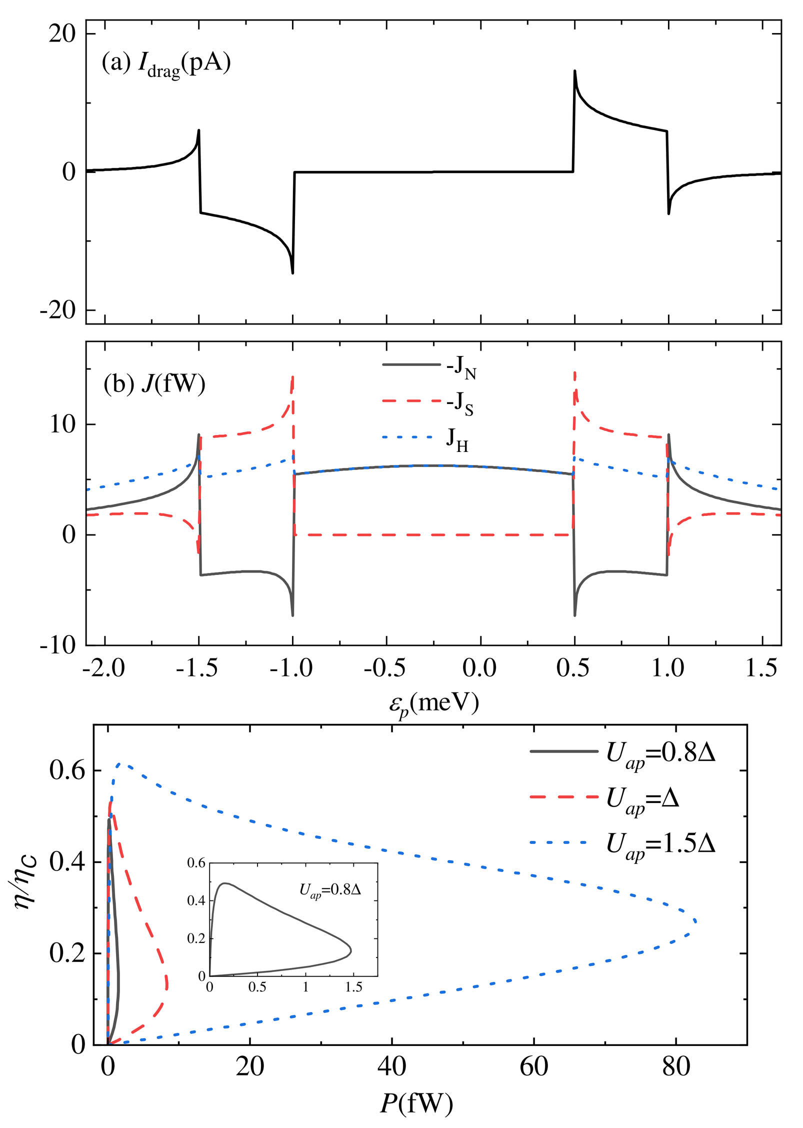
  $$\Gamma^{p,N}_{1n,0n} = \Gamma_N f_{Nn} \qquad \Gamma^{p,S}_{1n,0n} = \Gamma^{\rm qp}_{Sn} f_{Sn} = \Gamma_S \nu_n f_{Sn} \qquad \Gamma^{a,l}_{n1,n0} = \Gamma_\beta f_{ln}$$

  Here, $f_{ln} = f_l(\varepsilon_p + nU_{ap})$ ($l = N,S$), and $f_{ln} = f_l(\varepsilon_a + nU_{ap})$ ($l = H$).
  ν_n: density of states in S.

☞ Nonlocal current (V=0): $I_{\rm drag} \propto (\nu_0 - \nu_1)(f_0 - f_1)\Gamma_N\Gamma_S\Gamma_a$

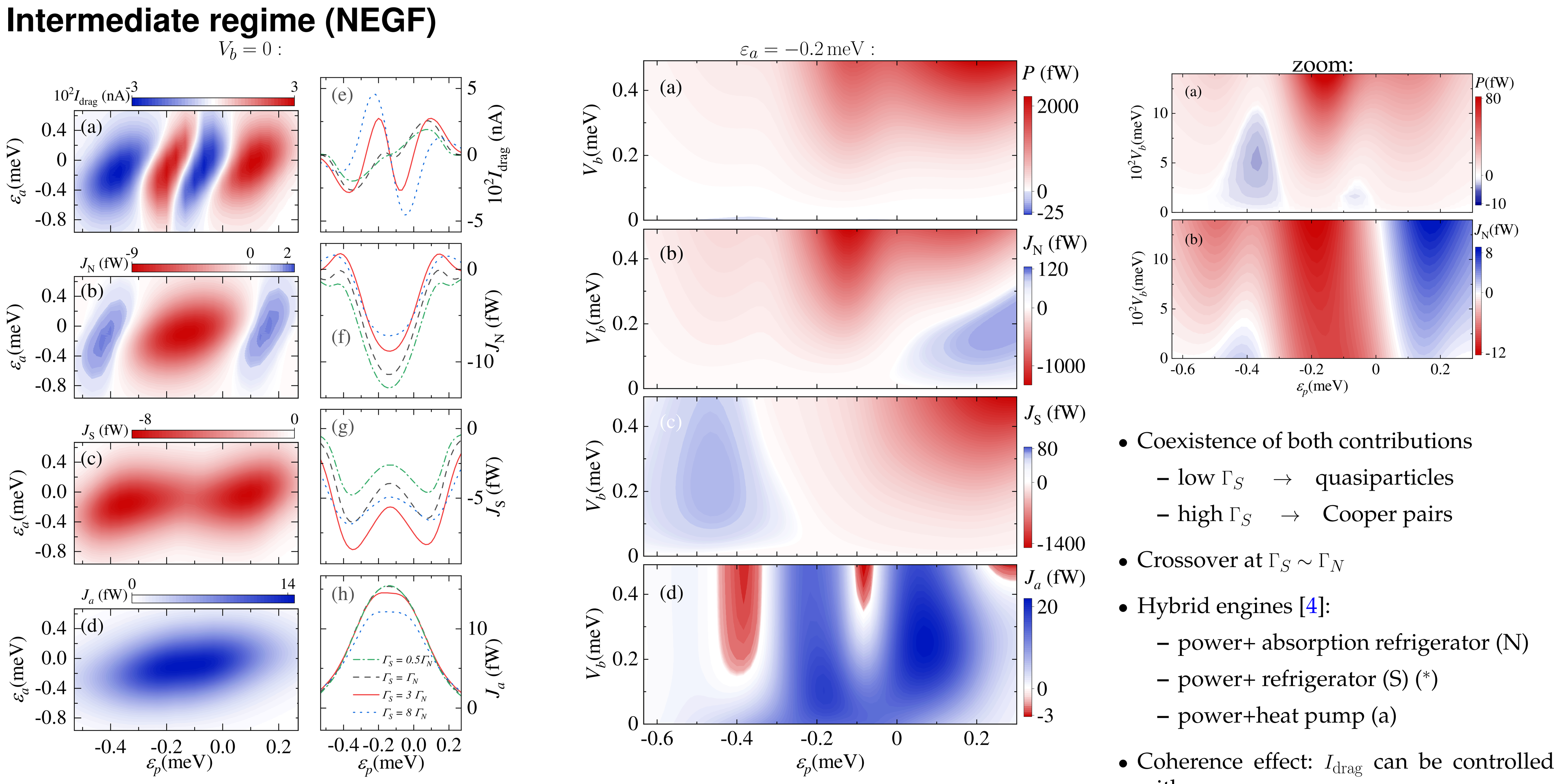
☞ The sign of the current is given by $\nu_0 - \nu_1$, (again) tuned with gates.





☞ The gap determines the sign of the current (opposite to the Andreev-Coulomb contribution)!

Intermediate regime (NEGF)



Parameters: $\Delta = 0.2\,$meV, $k_{\rm B}T = 0.1\,$meV, $k_{\rm B}T_H = 0.2\,$meV, $\Gamma_H = \Gamma_N = 0.01\,$meV, $\Gamma_S = 0.03\,$meV, $U_p = 1.6U_{ap} = 0.16\,$meV.

- Coexistence of both contributions
  - low Γ_S → quasiparticles
  - high Γ_S → Cooper pairs
- Crossover at $\Gamma_S \sim \Gamma_N$
- Hybrid engines [4]:
  - power+ absorption refrigerator (N)
  - power+ refrigerator (S) (*)
  - power+heat pump (a)
- Coherence effect: $I_{\rm drag}$ can be controlled with ε_a.