Geometric energy transport and refrigeration with driven quantum dots

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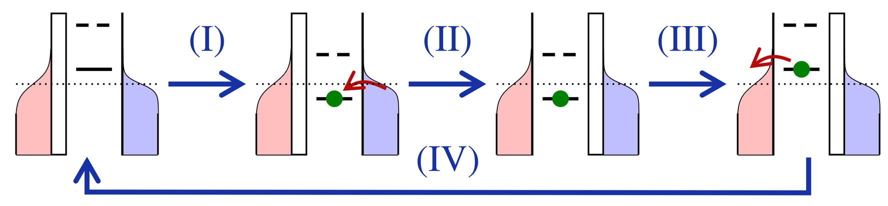
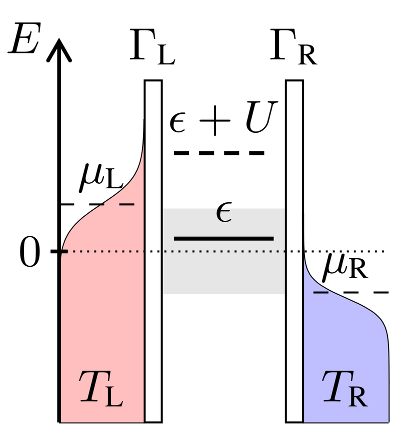
Motivations Single-level quantum dots are simple systems but with a rich physics, e.g. strong many-body interaction, to understand the characteristics of energy pumping.

- Spectroscopy: Characterization of quantum devices, new insights for AC-driving and energy transport
- Thermodynamics: Quantum dots are relevant for cyclic thermal machines as they can be time-dependently driven

Pumping: transport across the quantum dot due to the slow periodic modulation of system parameters



1. Single-level quantum dot coupled to two reservoirs



$\mu_{L/R} = \pm\frac{V_b}{2}$, $T_R = T$, $\Gamma_{L/R} = \Gamma(1\pm\Lambda)$
$(k_B \equiv \hbar \equiv e \equiv 1)$

Dot: $H = \epsilon(\hat{N}_\uparrow + \hat{N}_\downarrow) + U\hat{N}_\uparrow\hat{N}_\downarrow$, $|\rho) = (P_0, P_1, P_2)^T$

Occupation basis: $|0) = |0\rangle\langle 0|$, $|1) = \frac{|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|}{2}$, $|2) = |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$

Master equation (weak coupling $\hbar\Gamma_\alpha \ll k_B T_\alpha$)

$\partial_t|\rho(t)) = W|\rho(t))$ with kernel $W = -\gamma_p|p)(p'| - \gamma_c|c)(c'|$

stationary state $|z)$, parity mode $|p)$, charge mode $|c)$

Fermionic duality

$W^\dagger = -2\Gamma\mathcal{I} + \mathcal{P}W^i\mathcal{P}$, W^i kernel of **dual model**[1] with inverted energies $(\epsilon, U, \mu_\alpha, \Gamma_\alpha, T_\alpha) \mapsto (-\epsilon, -U, -\mu_\alpha, \Gamma_\alpha, T_\alpha)$

2. Slow periodic driving

$R_i(t) = \bar{R}_i + \delta R_i \sin(\Omega t + \phi_i)$ with $R_i \in \{\epsilon/T, U/T, V_b/T, \Lambda, T_L/T\}$, $i = 1, 2$

Slow driving $\Omega\delta R_i \ll \Gamma \Rightarrow$ time-dependent kernel $W(t)$

Expansion in orders of Ω/Γ

$|\rho) = |\rho^{(0)}) + |\rho^{(1)}) + \ldots \Rightarrow |\rho^{(0)}) = |z(t))$ and $|\rho^{(1)}) = \frac{1}{W}\partial_t|z(t))$

pseudo-inverse $\frac{1}{W} = -\frac{1}{\gamma_p}|p)(p'| - \frac{1}{\gamma_c}|c)(c'|$

Geometric approach

$|\rho^{(1)}) = \frac{1}{W}\nabla_{\mathbf{R}}|z)\cdot\partial_t\mathbf{R}$ and $\frac{1}{W}\nabla_{\mathbf{R}}|z) = \mathbf{x}_c|c) + \mathbf{x}_p|p)$ $\quad \mathbf{R}(t) = (R_1(t), R_2(t), 0)^T$

$\Rightarrow$ Time-dependent driving excite charge and parity modes

3. Transport quantities for the observable $\mathcal{O}$ from the dot into the lead α

Stationary transport

Current: $I^{(0)}_{\mathcal{O},\alpha} = (\mathcal{O}|W^\alpha|z)$

Cycle contribution: $\Delta\mathcal{O}^{(0)}_\alpha = \int_0^{\frac{2\pi}{\Omega}} I^{(0)}_{\mathcal{O},\alpha}(t)dt \sim \frac{\Gamma}{\Omega}$

First order correction

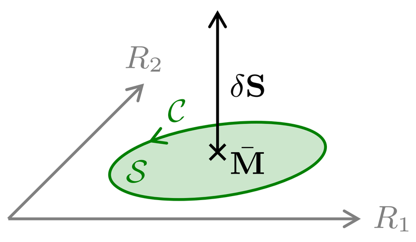
Current: $I^{(1)}_{\mathcal{O},\alpha} = (\mathcal{O}|W^\alpha\frac{1}{W}\nabla_{\mathbf{R}}|z)\cdot\partial_t\mathbf{R}$

Cycle contribution:

$$\Delta\mathcal{O}^{(1)}_\alpha = \int_0^{\frac{2\pi}{\Omega}} I^{(1)}_{\mathcal{O},\alpha}(t)dt$$

Geometric approach $= \oint_{\mathcal{C}} \mathbf{A}_{\mathcal{O},\alpha}(\mathbf{R})\cdot d\mathbf{R}$

Stokes theorem $= \iint_{\mathcal{S}} \mathbf{B}_{\mathcal{O},\alpha}(\mathbf{R})\cdot d\mathbf{S} \simeq \mathbf{B}_{\mathcal{O},\alpha}(\mathbf{R})\cdot\delta\mathbf{S}$



Geometric shape of transport quantity $\mathcal{O}$

- Landsberg geometric connection (vector potential)
$\mathbf{A}_{\mathcal{O},\alpha}(\mathbf{R}) = (\mathcal{O}|W^\alpha\frac{1}{W}\nabla_{\mathbf{R}}|z) = \mathbf{A}^C_{\mathcal{O},\alpha}(\mathbf{R}) + \mathbf{A}^P_{\mathcal{O},\alpha}(\mathbf{R})$
- Pumping curvature (pseudo-magnetic field)
$\mathbf{B}_{\mathcal{O},\alpha}(\mathbf{R}) = \nabla_{\mathbf{R}}\times\mathbf{A}_{\mathcal{O},\alpha}(\mathbf{R}) = \mathbf{B}^C_{\mathcal{O},\alpha}(\mathbf{R}) + \mathbf{B}^P_{\mathcal{O},\alpha}(\mathbf{R})$

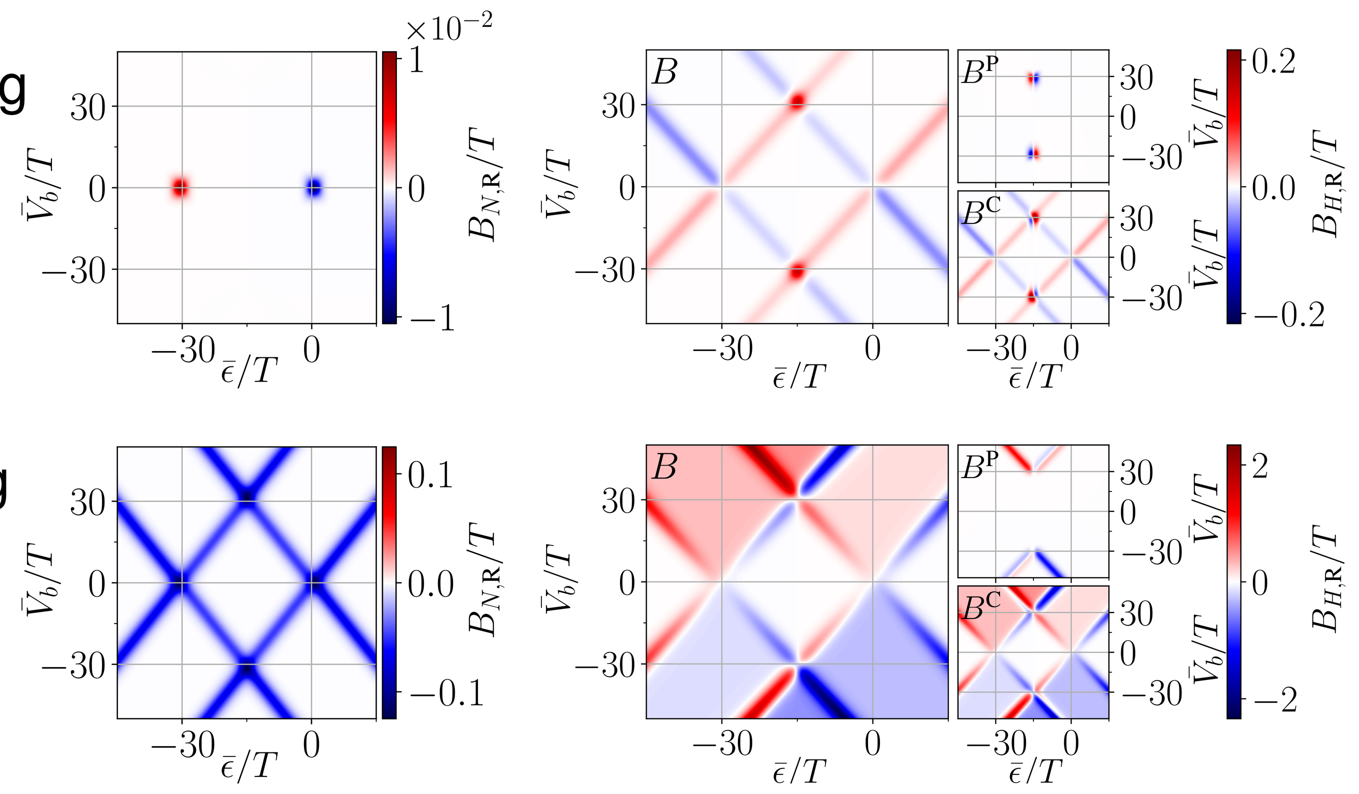
Duality + Geometry $\Rightarrow$ Insightful analytical expressions

4. Characteristics of charge and energy pumping

$\bar{\Lambda} = 0, \bar{T}_L = T, |\bar{U}| = 30T$

Repulsive onsite interaction $(U > 0)$

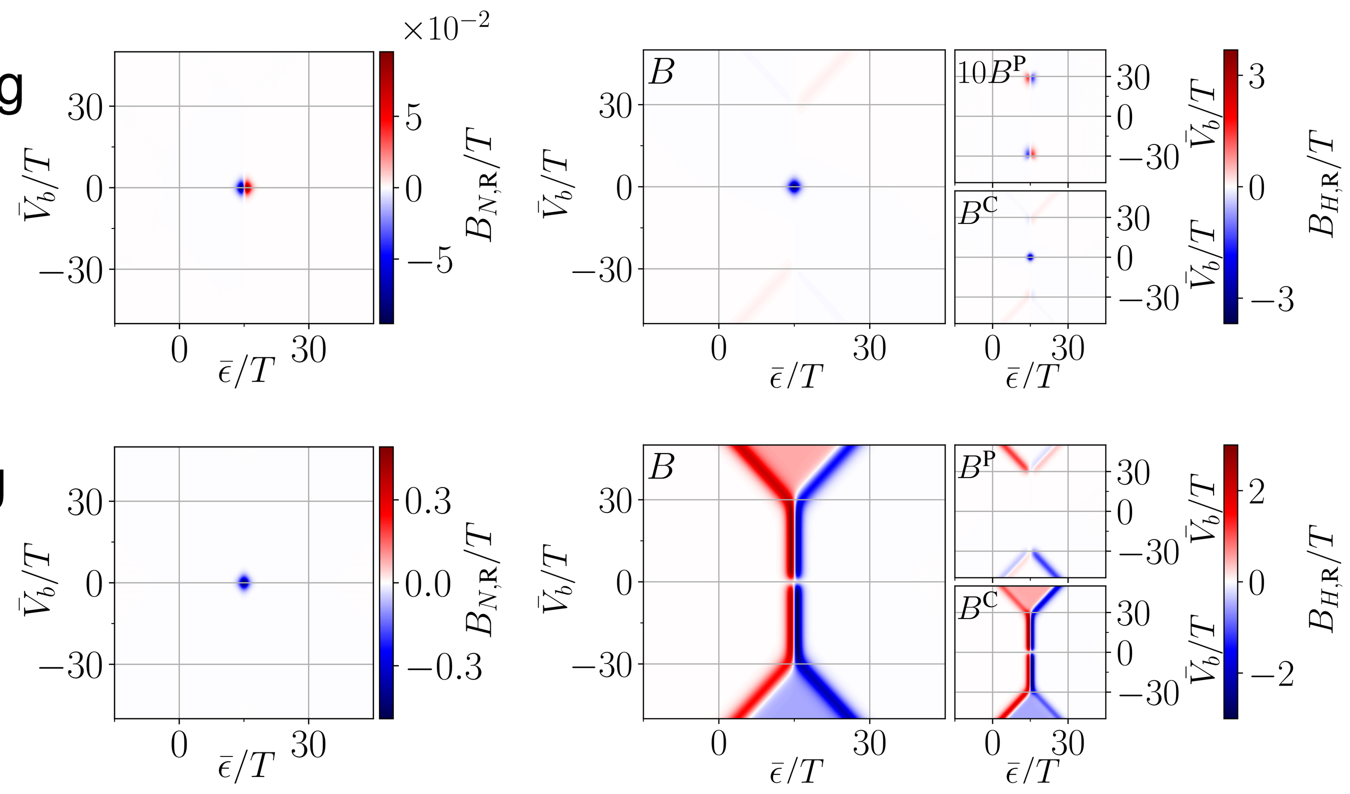
- (ϵ, V_b)-driving
- (ϵ, Λ)-driving



$\Rightarrow$ Complementarity of charge and energy pumping
$\Rightarrow$ Many-body effects only at high bias

Attractive onsite interaction $(U < 0)$[2–5]

- (ϵ, V_b)-driving
- (ϵ, Λ)-driving



$\Rightarrow$ Similar parity contribution at high bias
$\Rightarrow$ Special feature: two-particle resonance

5. Thermodynamics: refrigerator

$V_b = 0, \delta T = T_L - T_R > 0$

Driving scheme: $\begin{cases}\epsilon(t) = \bar{\epsilon} + \delta\epsilon\sin(\Omega t)\\ \Lambda(t) = \delta\Lambda\sin(\Omega t + \phi)\end{cases}$

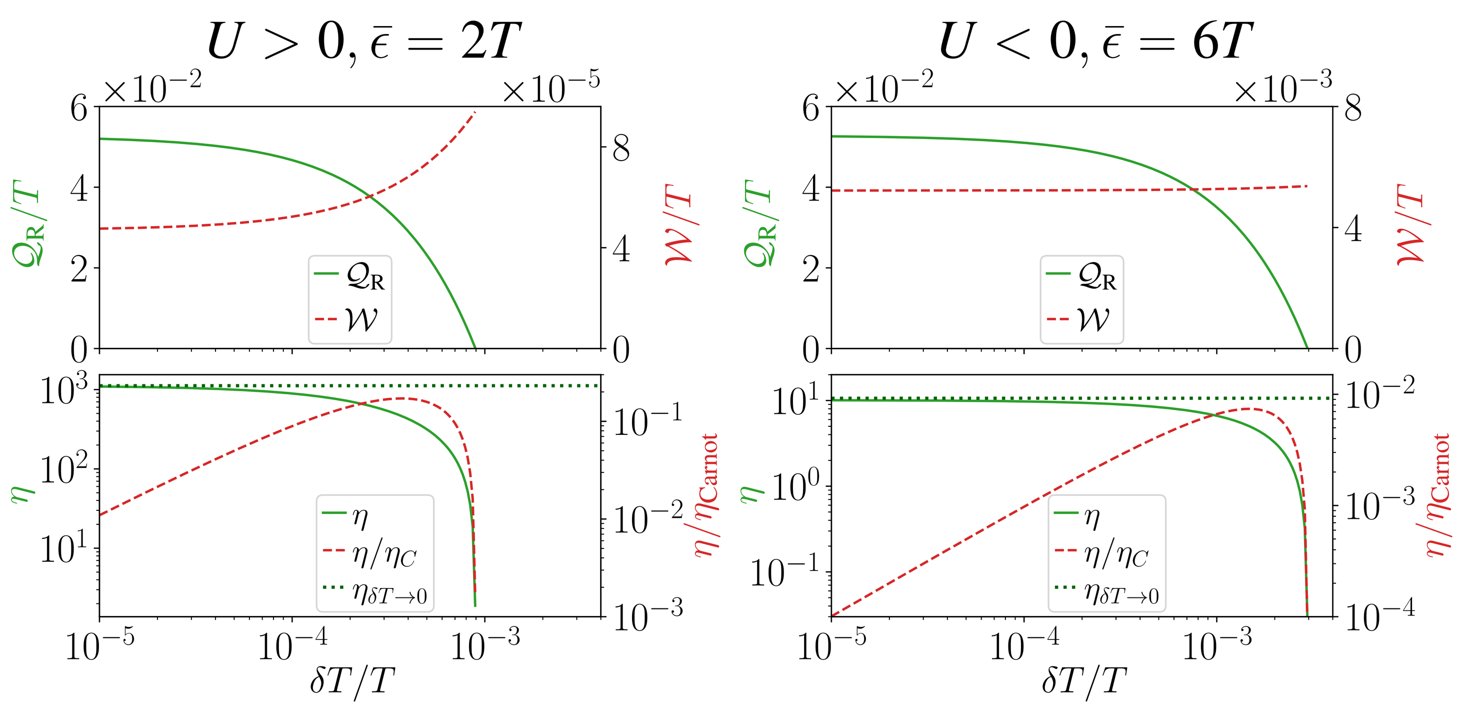
Heat: $\mathcal{Q}_R = \int_0^{2\pi/\Omega} dt\, I_{HR}(t) \underset{\text{adia.}}{\simeq} \mathcal{Q}^{(0)}_R + \mathcal{Q}^{(1)}_R$

Work: $\mathcal{W} = \int_0^{\frac{2\pi}{\Omega}} dt\,(\partial_t H|\rho(t)) \underset{\text{adia.}}{\simeq} \mathcal{W}^{(1)} + \mathcal{W}^{(2)}$

Efficiency:

$$\eta = \left|\frac{\mathcal{Q}_R}{\mathcal{W}}\right| \underset{\delta T\to 0}{\longrightarrow} |\sin(\phi)|\frac{\Gamma(f(\epsilon) + 1 - f(\epsilon+U))|\mathcal{E}^{eq}|\delta\Lambda}{2\Omega\delta\epsilon}$$

Seebeck energy $\mathcal{E}^{eq} = TS_{\text{Seebeck}}$, Fermi function $f(E)$



Parameters: $|U| = 10T$, $\Gamma = 10^{-2}T$, $\Omega = 10^{-2}\Gamma$, $\delta\epsilon = 10^{-1}T$, $\delta\Lambda = 1$, $\phi = \pi/2$

$\Rightarrow$ Sizable efficiency, up to $\sim 15\%$ of η_{Carnot}, at finite cooling power
$\Rightarrow$ $U > 0 \to$ larger efficiency
$\Rightarrow$ $U < 0 \to$ operation at larger temperature bias

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