Ground-state cooling of mechanical vibrations in a hybrid nano-electromechanical device

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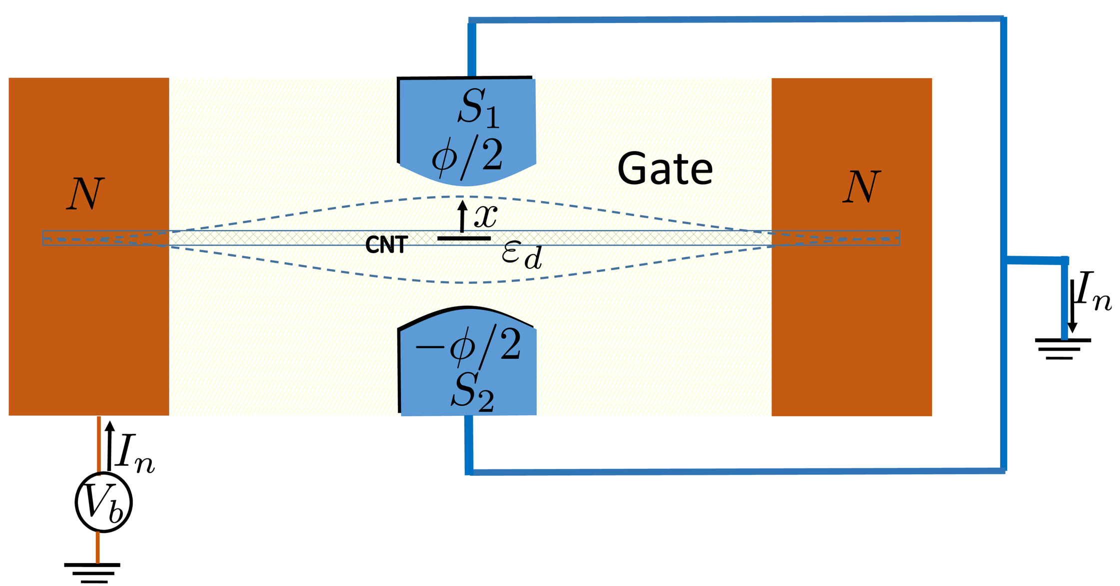
Abstract

We consider a nano-electromechanical (NEM) weak link composed of a carbon nanotube suspended above a trench in a normal metal electrode and positioned in a gap between two superconducting (SC) leads. The nanotube is treated as a movable single-level quantum dot (QD) in which the position-dependent superconducting order parameter is induced as a result of Cooper pair tunneling. We show that in such a system, self-sustained bending vibrations can emerge if a bias voltage is applied between normal and superconducting electrodes. The occurrence of this effect crucially depends on the direction of the bias voltage and the relative position of the quantum dot level. We also demonstrate that the nanotube vibrations strongly affect the dc current through the system, a characteristic that can be used for thedirect experimental observation of the predicted phenomenon.

Model of NEM device



$H_d = \sum_\sigma \varepsilon_d d^\dagger_\sigma d_\sigma, \quad \{d^\dagger_\sigma, d_{\sigma'}\} = \delta_{\sigma,\sigma'}$ $\qquad H_v = \frac{\hbar\omega}{2}\left(\hat{x}^2+\hat{p}^2\right),$

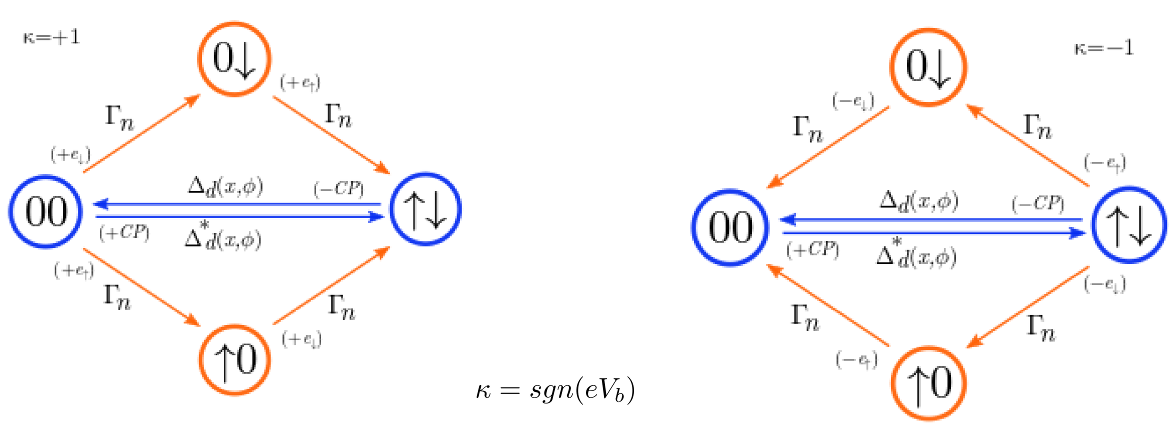
$H^n_l = \sum_{k\sigma}(\varepsilon_k - eV_b)a^\dagger_{k\sigma}a_{k\sigma},$ $\qquad H^s_l = \sum_{kj\sigma}\left(\varepsilon_{kj}c^\dagger_{kj\sigma}c_{kj\sigma} - \Delta_s(e^{\imath\phi_j}c^\dagger_{kj\uparrow}c^\dagger_{-kj\downarrow} + \text{H.c.}\right),$

normal lead — *superconducting leads (BCS)*

$H^n_t = \sum_{kj\sigma} t^n_{kj}(a^\dagger_{kj\sigma}d_\sigma + \text{H.c.}),$ $\qquad H^s_t = \sum_{kj\sigma} t^s_{kj}(x)(c^\dagger_{kj\sigma}d_\sigma + \text{H.c.});$

tunneling between the dot and the normal lead — *tunneling between the dot and SC leads with the position-dependent tunneling amplitude* $t^s_{kj}(x) = t^s_0 e^{\mp x/(2\lambda)}.$

Electronic transitions



$\kappa = sgn(eV_b)$

Density matrix approximation

$$\imath\partial_t\rho = [H,\rho] + \mathcal{L}_\gamma\{\rho\},$$

$\Delta_s \gg eV_b \gg \varepsilon_d, k_BT$ $\qquad \rho = \rho^n_l \otimes \rho^s_l \otimes \rho_d \otimes \rho_m.$

$$-\imath\left[H^{eff}_d, \rho_{dm}\right] + \mathcal{L}_n\{\rho_d\} + \mathcal{L}_\gamma\{\rho_m\} = 0,$$

$H^{eff} = H_d + \Delta_d(x,\phi)d_\downarrow d_\uparrow + \Delta^*_d(x,\phi)d^\dagger_\uparrow d^\dagger_\downarrow$

equilibrium thermal environment

$\mathcal{L}_\gamma\{\rho_m\} = -\gamma\left(n_B + \frac{1}{2}\right)[x,[x,\rho_m]] - \imath\frac{\gamma}{2}[x,\{p,\rho_m\}].$

$\Delta_d(x,\phi) = \Delta_d\cosh\left(\frac{x}{\lambda} + \imath\frac{\phi}{2}\right)$

SC proximity effect

non-equilibrium electronic environment

$$\mathcal{L}_n\{\hat\rho_d\} = \Gamma_n\sum_\sigma \begin{cases} 2d^\dagger_\sigma\hat\rho_d d_\sigma - \{d_\sigma d^\dagger_\sigma, \hat\rho_d\}, & \kappa = +1; \\ 2d_\sigma\hat\rho_d d^\dagger_\sigma - \{d^\dagger_\sigma d_\sigma, \hat\rho_d\}, & \kappa = -1; \end{cases}$$

$\kappa = sgn(eV_b)$

$$W(x,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\xi e^{-\imath p\xi}\langle x + \xi/2|\rho_m|x - \xi/2\rangle.$$

Wigner function representation

Low Temp. Phys., **48**, 476 (2022).

see also,

ArXiv: 2112.00210

New J. Phys., **24**, 033008 (2022).

and ArXiv: 2202.08009

Ground-state cooling

$$P_n = \frac{2\beta}{1+\beta}\left(\frac{1-\beta}{1+\beta}\right)^n, \qquad \beta = \mathcal{D}_1/2\mathcal{D}_2,$$

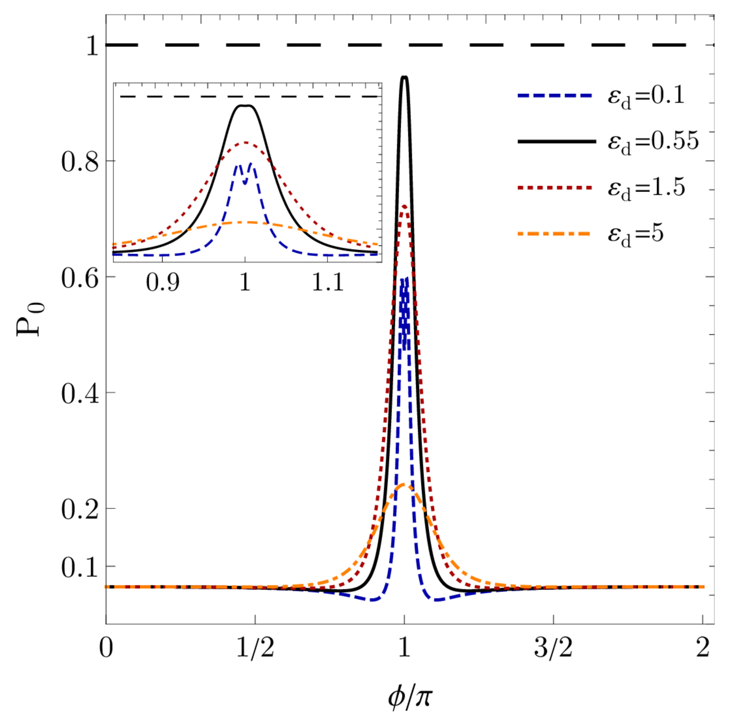
$$\mathcal{D}_1 = -\kappa\frac{\Delta^2_d\Gamma_n\varepsilon_d}{\lambda^2 D_1}\sin^2(\phi/2) + \gamma,$$

$$\mathcal{D}_2 = \frac{\Delta^2_d\Gamma_n C}{\lambda^2 D_1}\sin^2(\phi/2) + \gamma\left(n_B + 1/2\right),$$

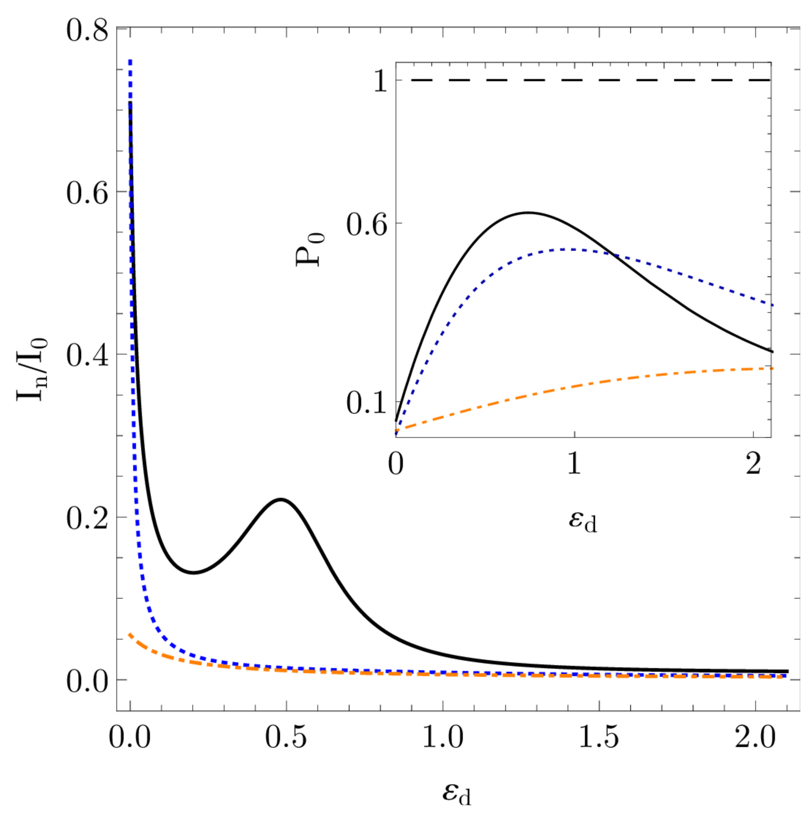
$$D_1 = (D - 1/4)^2 + \Gamma^2_n$$

$$C = \frac{(D + 1/4)\left(D + \varepsilon^2_d + \Gamma^2_n\right) - 4\Delta^2_d\Gamma^2_n\cos^2(\phi/2)}{4D}$$

$$D = \varepsilon^2_d + \Gamma^2_n + \Delta^2_d\cos^2(\phi/2)$$



Electric current



$$I_n = I_0\frac{\Delta^2_d\cos^2(\phi/2)}{\Gamma^2_n + \varepsilon^2_d + \Delta^2_d\cos^2(\phi/2)} + \mathcal{O}\left(1/\lambda^2\right)$$

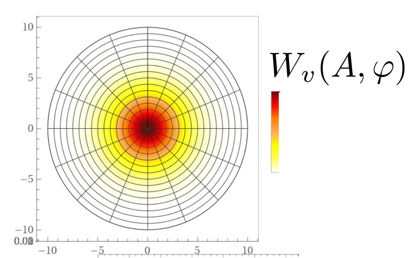
$$I_0 = e\Gamma_n$$

$\underline{\phi = \pi}:$

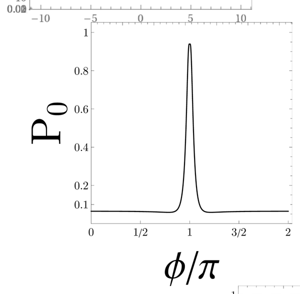
$$I_n = I_0\left(\frac{\Delta_d}{\lambda}\right)^2\frac{\left(\Gamma^2_n + \varepsilon^2_d + 1/4\right)\langle x^2\rangle + \varepsilon_d/2}{\left(\Gamma^2_n + \varepsilon^2_d - 1/4\right)^2 + \Gamma^2_n}$$

$$\langle x^2\rangle = (2\beta)^{-1}$$

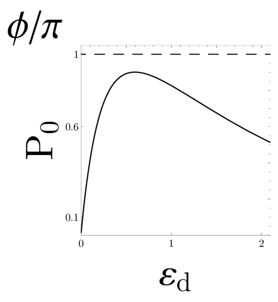
Summary



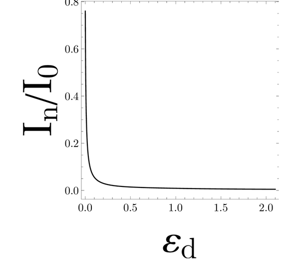
- The stationary state of the mechanical subsystem has a Boltzmann form if a bias voltage is applied between the normal and superconducting leads.



- In the *cooling* regime the probability to find the system in the ground state is near the maximum and crucially depends on the superconducting phase difference and the relative position of the QD energy level.



- The direct *electric current* behavior mirrors the stationary state of the system. This can be served for an experimental detection of the predicted effects.



Acknowledgments

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