Bayesian network statistics

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Motivation

- ➔ How can we estimate the statistics for thermodynamic quantities without jeopardizing quantum coherences and correlations?
- ➔ How do initial correlations contribute to such statistics?
- ➔ Are there different possible estimations with different precision?

Background

Our estimations are made with the use of Bayesian Networks (BN)[1] in which we infer probabilities of events conditioned on previous events, eliminating the need for measurements in the construction of the distributions.

BNs and our setup

- ☛ Our setup consists of a global system with two parties A and B interacting via an unitary U;
- ☛ If the global system starts with an initial state $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, then the BN infer that the probability of the subsystem A start at the state $|a_0\rangle$ and evolve to $|a_t\rangle$ after the unitary evolution is

$$\mathcal{P}(a_t, a_0) = \sum_i p_i P(a_0|\psi_i) P(a_t|\psi_{i_t}),$$

where $P(a_0|\psi_i) = |\langle a_0|\psi_i\rangle|^2$ and $P(a_t|\psi_t) = |\langle a_t| U |\psi_i\rangle|^2$.

Results

➤ Given an observable $\mathcal{A}$ in the subsystem A, we obtained the first two moments of its variation $\Delta\mathcal{A}$

$$\langle\Delta\mathcal{A}\rangle = \mathrm{Tr}\left[\rho(U^\dagger \mathcal{A} U - \mathcal{A})\right]$$

and

$$\langle(\Delta\mathcal{A})^2\rangle = \mathrm{Tr}\left[\rho((U^\dagger\mathcal{A}U)^2 + A^2)\right] - 2\sum_i p_i \langle\psi_i| A |\psi_i\rangle \langle\psi_i| U^\dagger \mathcal{A} U |\psi_i\rangle,$$

from the characteristic function

$\mathrm{G}(k) = \sum_i p_i \langle\psi_i| e^{-ik\mathcal{A}} |\psi_i\rangle \langle\psi_i| e^{ikU^\dagger\mathcal{A}U} |\psi_i\rangle$.

■ Arbitrary ensemble choices in ρ infer different BN probability distributions. But the average of $\Delta\mathcal{A}$ is independent of the ensemble choice!

Heat in the bosonic case

➤ The parties A and B are single bosonic modes, U is the Beam Slitter (BS) interaction and $\mathcal{A}$ is the local Hamiltonian of A, then $\Delta\mathcal{A} = Q$ is the heat exchanged by A. If initially the global covariance matrix is

$$\sigma_{AB} = \begin{pmatrix} a & 0 & c_+ & 0 \\ 0 & a & 0 & c_- \\ c_+ & 0 & b & 0 \\ 0 & c_- & 0 & b \end{pmatrix},$$

then the average heat will be

$$\langle Q\rangle = \omega[\sin^2(g)(b-a) + \sin(g)\cos(g)(c_+ + c_-)],$$

where g is the BS strength.

➤ Choosing the ensemble of coherent states for representing the density matrix, we arrive at a larger variance distribution than choosing the ensemble of eigenvectors of ρ.

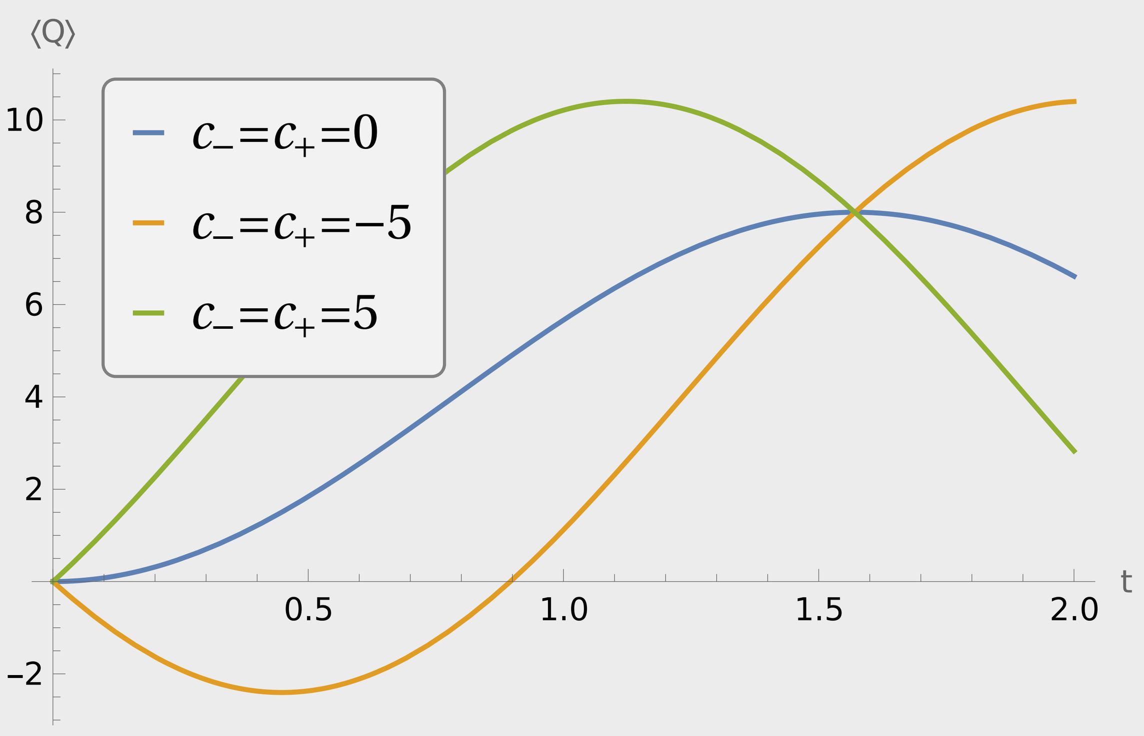

Figure 1: Average of heat for different choices of correlations (all with $a = 3$ and $b = 11$). For certain combinations of initial correlations, described by c_+ and c_-, we have the inversion of the heat flux.[2]

Conclusions and future perspectives

✔ The statistics sustain effects of the initial correlations, reproducing descriptions such as the inversion of heat flux;[2]

✐ The relation of higher moments for different ensemble choices remain unclear. The analysis of the bosonic heat case seems to suggest that the ensemble of eigenvectors of ρ is the one with least variance.

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References

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[2] K. Micadei, J. P. S. Peterson, A. M. Souza, R. S. Sarthour, I. S. Oliveira, G. T. Landi, T. B. Batalhão, R. M. Serra, and E. Lutz, Reversing the direction of heat flow using quantum correlations, Nature Communications **10** (2019).