Third law of thermodynamics and the scaling of quantum computers

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Problem definition

An implicit assumption in quantum computation is to **initialize the qubits register** in the computational state $|00\dots0\rangle$ before applying a general unitary. In this work [1], we wanted to investigate how the fidelity for preparing a multi-qubit (pure) state is affected by the **third law of thermodynamics**.

By definition, the $|00\dots0\rangle$ state is a **zero temperature state**. However, from the the third law of thermodynamics, we are bound to prepare states that have an arbitrary small, but **finite**, temperature [2, 3]. Thus, we assume that the *real* initial state of the system is the thermal state $\rho_0 = (e^{-\beta H}/Z)^{\otimes N}$ (Z is the appropriate partition function) that reads explicitly:

$$\rho_0 \equiv \left(\frac{1}{1+e^{-\beta\Delta E}} \begin{pmatrix} e^{-\beta\Delta E} & 0 \\ 0 & 1 \end{pmatrix} \right)^{\otimes N} \tag{1}$$

where β is the effective inverse temperature of the initial (prepared) state and ΔE is the energy difference between the states $|0\rangle$ and $|1\rangle$ of the single qubits. This corresponds to have a single-qubit **initialization error** $\eta = 1-(1+e^{-\beta\Delta E})^{-1}$.

What we want to understand is how this effect, that originates from a fundamental thermodynamical constraint, has repercussions on the initialization fidelity of the multi-qubit register and on the subsequent quantum computation. In particular we want to find a scaling law for the initialization fidelity $\mathcal{F}(\rho_0, \sigma_0)$ between our finite-temperature state ρ_0 and our target initialization state $|00\dots0\rangle = \sigma_0$.

Main result

The **scaling** of $\mathcal{F}(\rho_0, \sigma_0)$ as a function of the parameters N and β is

$$\mathcal{F}(\rho_0, \sigma_0) = \left(1+e^{-\beta\Delta E}\right)^{-N} \tag{2}$$

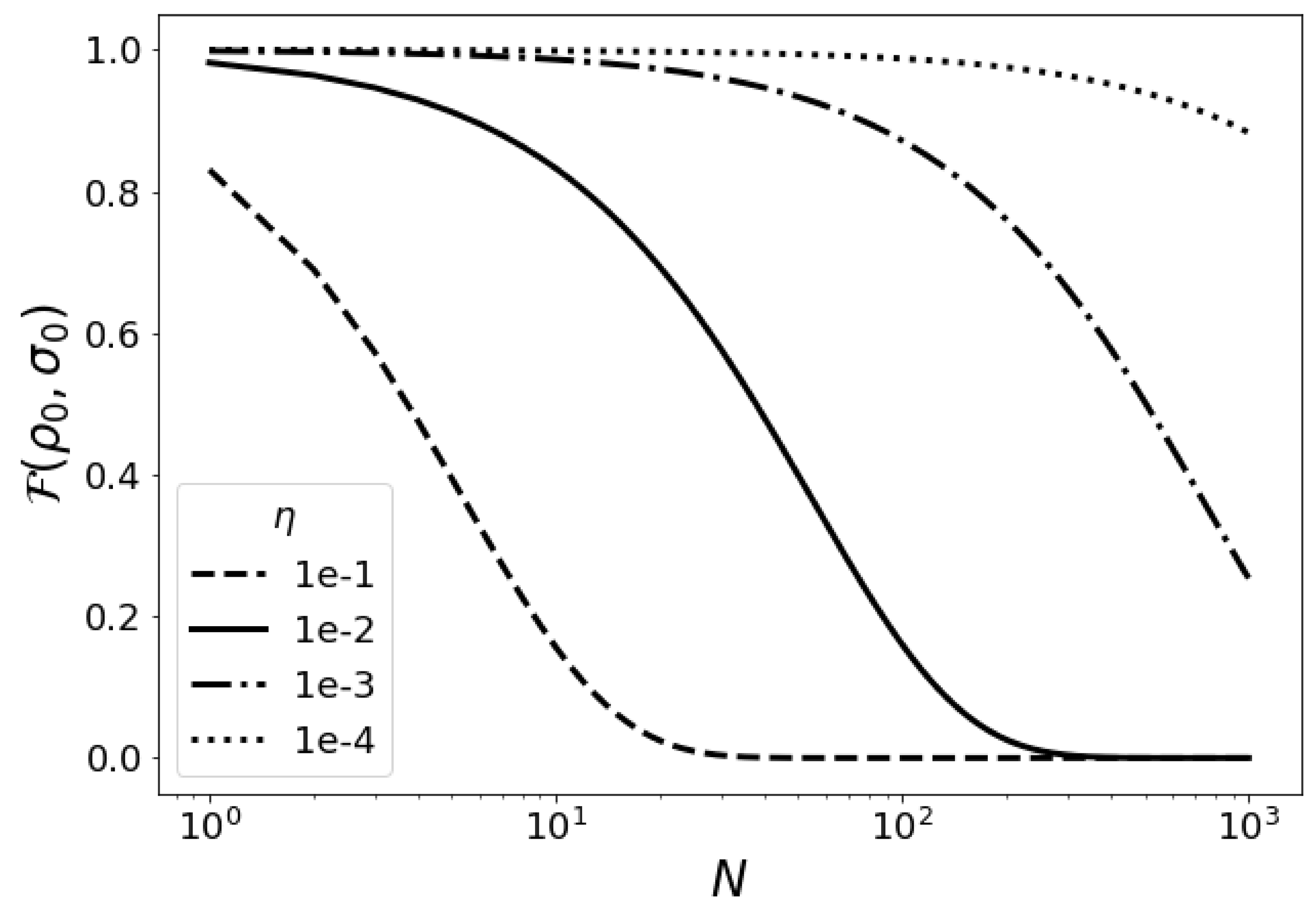
This shows that **any nonzero value** for the initial inverse temperature β of the real state ρ_0 can end up hindering the scaling (i.e., $N \to \infty$) of the considered quantum circuit or algorithm.

The reason behind this result being so general lies again in the thermodynamic considerations behind the third law, and thus in the divergent cost of attaining a perfect pure state (i.e., with $\beta \to \infty$). In fact, it now becomes clear that the issue of scaling quantum computers regards two **competing limits**:

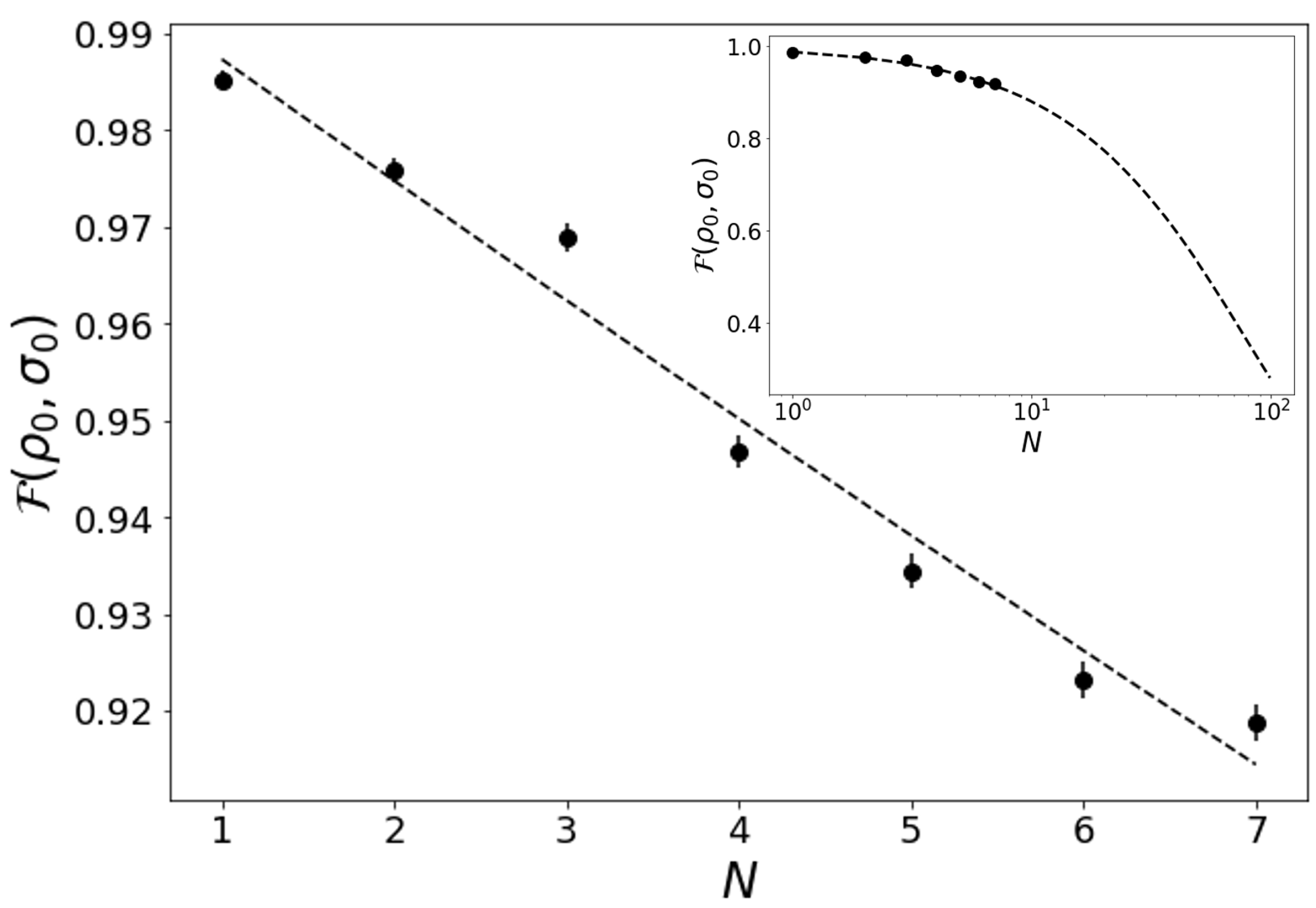
$$\lim_{N\to\infty} \lim_{\beta\to\infty} \mathcal{F}(\rho_0, \sigma_0) = 1 \tag{3}$$

$$\lim_{\beta\to\infty} \lim_{N\to\infty} \mathcal{F}(\rho_0, \sigma_0) = 0\,. \tag{4}$$

Scaling Plot



Experiment – Results



Experiment – Setup

The scaling experiment realized consists in locally measuring the initial register state $|00\dots0\rangle$ immediately after its preparation, it was performed on an ibm-lagos 7 qubit processor.

By performing the experiment for different number N of qubits from 1 to 7, we obtain the results reported in the figure besides. From the figure one can observe that, while the **single-qubit** initialization fidelity is almost 99%, as the number of qubits increases the fidelity **drops significantly** to around 92%.

For a quantitative evaluation, we fit the value of $\beta\Delta E$ over the experimental data, getting a value of $\beta\Delta E = 4.35 \pm 0.03$ with a coefficient of determination $R^2 = 0.976$. The resulting curve, whose analytical expression is provided by Eq. (2), is plotted as the **dashed line** in the figure

Conclusions

From the assumption that an initialization of a qubit register will be subjected to a **finite temperature** (and thus to a finite error) we derived the following conclusions:

- A **scaling** form of the fidelity for multi-qubit initialization.
- The proof that this fidelity is a **upper bound** for every state preparation thereafter.
- An **experimental test** on a real quantum device, even if limited to 7 qubits.
- The fact that **thermodynamical bounds** will be crucial for the future of large-scale quantum computers.

Bibliography

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3. Ticozzi, F. *et al.* Quantum resources for purification and cooling: fundamental limits and opportunities. *Scientific reports* **4,** 1–7 (2014).