Fast Squeezing and Cooling of Quantum Systems

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Squeezing in arbitrary time

Preparation in arbitrary time

$$\sigma_0 = e^{-\beta_0 H_0}/\mathrm{Tr}(e^{-\beta_0 H_0}) \quad \Rightarrow \quad \sigma_t = \frac{1}{Z_t} S_{r,\phi} e^{-\frac{\varepsilon_t}{\varepsilon_0}\beta_0 H_0} S^\dagger_{r,\phi},$$

Harmonic Oscillator Hamiltonian

$$H_0 = \hbar\omega_0(a_0^\dagger a_0 + 1/2)$$

$$a_t = \sqrt{\frac{m\omega_t}{2\hbar}}\hat{x} + i\sqrt{\frac{1}{2\hbar m\omega_t}}\hat{p}.$$

Squeezing Operator

$$S_{r,\phi} = \exp\left(\frac{r_t}{2}\left(e^{-i\phi_t}a_0^2 - e^{i\phi_t}a_0^{\dagger 2}\right)\right)$$

Dynamics?

$$\dot{\sigma}_t = -\frac{i}{\hbar}[H_{\mathrm{GHO}} + H_{\mathrm{cd}}, \sigma_t]$$

$$H_{\mathrm{GHO}} = \hbar\omega_t(A_t^\dagger A_t + 1/2).$$

$$H_{\mathrm{cd}} = \hbar\frac{\dot{\phi}_t}{2}(A_t^\dagger A_t + \frac{1}{2} - (a_0^\dagger a_0 + \frac{1}{2})) + i\hbar\frac{\dot{r}_t}{2}(a_0^2 e^{-i\phi_t} - a_0^{\dagger 2}e^{i\phi_t}).$$

$$A_t \equiv S_{r,\phi}a_0 S^\dagger_{r,\phi} = \cosh r_t a_0 + e^{i\phi_t}\sinh r_t a_0^\dagger.$$

Case $\phi = 0$

$$H_{\mathrm{CD}}^{(0)} = i\hbar\frac{\dot{r}_t}{2}(a_0^2 - a_0^{\dagger 2})$$

Case $\phi \neq 0$

$$\tilde{\sigma}_t = U_\phi \sigma_t U_\phi^\dagger.$$

$$U_\phi = e^{-i\frac{\phi_t}{2}(a_0^\dagger a_0 + \frac{1}{2})}.$$

$$\frac{d\tilde{\sigma}_t}{dt} = \frac{1}{i\hbar}[i\hbar\frac{\dot{r}_t}{2}(a_0^2 - a_0^{\dagger 2}), \tilde{\sigma}_t].$$

$$\phi_f = -2\omega_0 t_f.$$

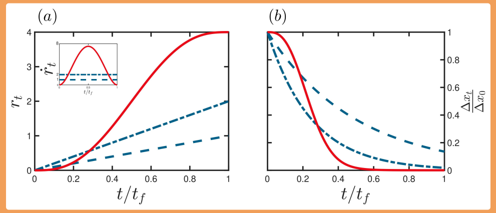

Fig. 1. Evolution of (a) **Squeezing amplitude** r_t as function of the process time with linear variation (blue curves) or through a controlled dynamics (red curve); and corresponding (b) **normalized variance in position**.

Experimental implementation

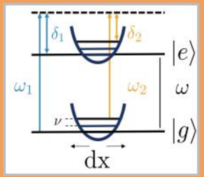

Fig. 2. Implementation of the two-photon Raman Hamiltonian with two detuned laser beams $\omega_1 - \omega_2 = 2\nu$.

Liouville Von Neumann equation

$$\frac{d\rho_t}{dt} = -i[\alpha_t a^2 + \alpha_t^* a^{\dagger 2}, \rho_t],$$

$$\alpha_t = (\eta_2 - \eta_1)^2 \frac{\Omega_1(t)\Omega_2(t)}{4\Delta} e^{i\Phi_t}.$$

Rabi Frequency

$$\hbar\Omega_l/2 = q\langle g|\hat{x}|e\rangle A_l(t).$$

Dephasing

$$\Phi_t = (\Phi_1 - \Phi_2).$$

Lamb-Dicke parameter

$$\eta_l = k_l\sqrt{\hbar/(2m\omega_0)}.$$

Delta-kick Cooling & Optimal Control

What is Delta-kick Cooling?

Expansion of time t_k — kick duration τ_k

$$\vec{f} = -m\omega_k^2\vec{r} \qquad \delta\vec{p} = \vec{f}\tau_k = -m\tau_k\omega_k^2\vec{r}$$

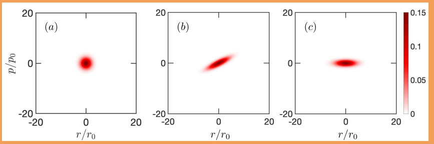

Fig.3. Wigner function of a thermal state of a quantum oscillator during a standard DKC protocol. (a) Initial state with inverse temperature $\beta_0 = 1/(\hbar\omega_0)$. (b) After the expansion time $t_k = 3/2\omega_0$, where ω_0 is the initial trap frequency. (c) Upon completion of the protocol, after the kick.

$$\tau_k\omega_k^2 = 1/t_k \quad (\triangle)$$

Kicked-Hamiltonian

$$H_k(t) = H(t) + \delta(t - t_k)\frac{1}{2}m\omega_k^2\sum_{i=1}^{N}\vec{r}_i^{\,2}.$$

Propagator

$$U_\delta(t,0) = U(t, t_k + \tau_k)e^{-i\tau_k\frac{m\omega_k^2}{2\hbar}\sum_{i=1}^{N}\vec{r}_i^{\,2}}U(t_k, 0),$$

Analytical approach to delta-kick cooling using scale-invariant dynamics

Scale-Invariant Hamiltonian

$$H(t) = \sum_{i=1}^{N}\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}m\omega(t)^2\vec{r}_i^{\,2}\right] + \sum_{i<j}V(\vec{r}_i - \vec{r}_j),$$

$$\Psi(t) = \frac{1}{b^{\frac{DN}{2}}}\exp\left[i\frac{m\dot{b}}{2\hbar b}\sum_{i=1}^{N}\vec{r}_i^{\,2} - i\int_0^t \frac{E(0)}{\hbar b(t')^2}dt'\right] \times\Psi\left(\frac{\vec{r}_1}{b},\ldots,\frac{\vec{r}_N}{b}, t=0\right),$$

Ermakov equation $\quad \ddot{b} + \omega(t)^2 b = \omega_0^2/b^3,$

New kick time $\quad \tau_k\omega_k^2 = \frac{\dot{b}(t_k)}{b(t_k)}.\ (\square)$

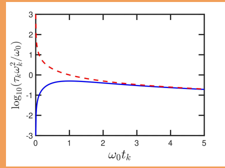

Fig.4. Comparison between the pulse parameters determined by the exact DKC relation (blue, solid line) and the long-time limit (red, dashed line).

Link with optimal control
How to modulate the frequency?

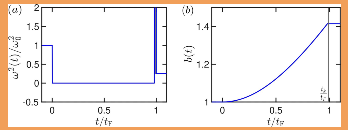

Fig.5. DKC after free expansion. (a) Time evolution of $\omega(t)^2/\omega_0^2$ during the process, with parameters $1/5\omega_k = 2\omega_F = \omega_0$. (b) Time evolution of $b(t)$.

$$\omega(t) = \begin{cases}\omega_0 & t \leq 0\\ 0 & 0 < t < t_k\\ \omega_k & t_k < t < t_k + \tau_k\\ \omega_F & t \geq t_F = t_k + \tau_k.\end{cases} \qquad t_k = \frac{1}{\omega_0}\sqrt{b_F^2 - 1 + \frac{1 - b_F^2}{b_F^2}\left(\frac{\omega_0}{\omega_k}\right)^2}, \quad \tau_k = \frac{1}{\omega_k}\arcsin\sqrt{\frac{b_F^2 - 1}{b_F^4\frac{\omega_k^2}{\omega_0^2} - 1}}. \quad (\star)$$

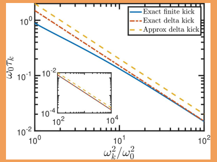

Fig.6. DKC with finite pulses. Comparison of the pulse parameters in the exact finite-pulse analysis ($\star$), the exact δ-kick treatment ($\square$) and long-time δ-kick approximation ($\triangle$), taking $\omega_F = \omega_0/2$.

References

[1] L.Dupays, A. Chenu, Shortcuts to Squeezed Thermal States, Quantum, 5, 449.
[2] L.Dupays, D.C Spierings, A.M. Steinberg, A. del Campo, Exact Delta Kick Cooling, Time-Optimal Control of Scale-Invariant Dynamics and Shortcuts to Adiabaticity Assisted by Kicks- arXiv preprint arXiv:2104.00999.