Bounds on skewness and kurtosis of steady state currents

arXiv:2204.08421 (2022)

Krzysztof Ptaszyński

Institute of Molecular Physics, Polish Academy of Sciences, M. Smoluchowskiego 17, 60-179 Poznań, Poland,
krzysztof.ptaszynski@ifmpan.poznan.pl

Main message

- Bounds on 3rd and 4th current moments (skewness and kurtosis) are derived or conjectured for several classes of open systems
- Their breaking reveals information about dynamics and thermodynamics of a transport setup

Definitions

- Integrated stochastic current

$$J_t = \int_0^t j(\tau) d\tau$$

- Current central moments

$$\langle j \rangle = \lim_{t\to\infty} \frac{\langle J_t \rangle}{t}$$

$$\langle \Delta j^n \rangle = \lim_{t\to\infty} \frac{\langle (J_t - \langle J_t \rangle)^n \rangle}{t}$$

- Skewness and kurtosis

$$\mathcal{S} = \frac{\langle \Delta j^3 \rangle}{\langle j \rangle}$$

$$\mathcal{K} = \frac{\langle \Delta j^4 \rangle}{\langle \Delta j^2 \rangle} - 3 \langle \Delta j^2 \rangle$$

Summary of results

1. For noninteracting fermionic systems kurtosis of the particle current obeys the relation

$$\mathcal{K}^p \in \left[-\frac{1}{2}, 1 \right]$$

Additionally, for junctions driven only by a single voltage

$$\mathcal{S}^p \in \left[-\frac{1}{2}, 1 \right]$$

2. For noninteracting bosonic systems kurtosis of the particle current j_p and the heat current j_h is always nonnegative

$$\mathcal{K}^p, \mathcal{K}^h \geq 0$$

For systems driven only by a single thermodynamic force (difference of bath temperatures or chemical potentials)

$$\mathcal{S}^p, \mathcal{S}^h \geq 0$$

3. In classical Markovian systems driven only by temperature differences

$$\mathcal{K}^h \geq 0$$

For systems driven by a single temperature difference

$$\mathcal{S}^h \geq 0$$

4. For unicyclic Markovian networks skewness and kurtosis of the winding number (i.e., number of rotations around the cycle) obey the relations

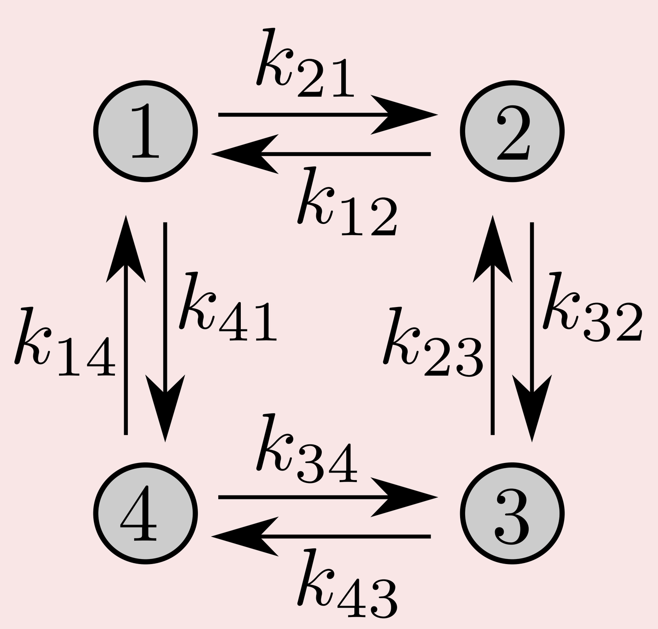
$$\mathcal{S} \in \left[-\frac{1}{16}, 1 \right]$$

$$\mathcal{K} \in \left[-\frac{\sqrt{5}+1}{10}, 1 \right]$$

$$\mathcal{K} - \mathcal{S} \in [-0.15, 0.465]$$

$$\mathcal{K} + \mathcal{S} \in \left[-\frac{8}{27}, 2 \right]$$

$$\mathcal{K} \times \mathcal{S} \in [-0.054, 1]$$



5. For classical Markovian systems [1]

$$\mathcal{S}_{\text{lin}} = \mathcal{K}_{\text{eq}} \geq 0$$

where $\mathcal{S}_{\text{lin}}$ – linear response skewness, $\mathcal{K}_{\text{eq}}$ – equilibrium kurtosis

Methods

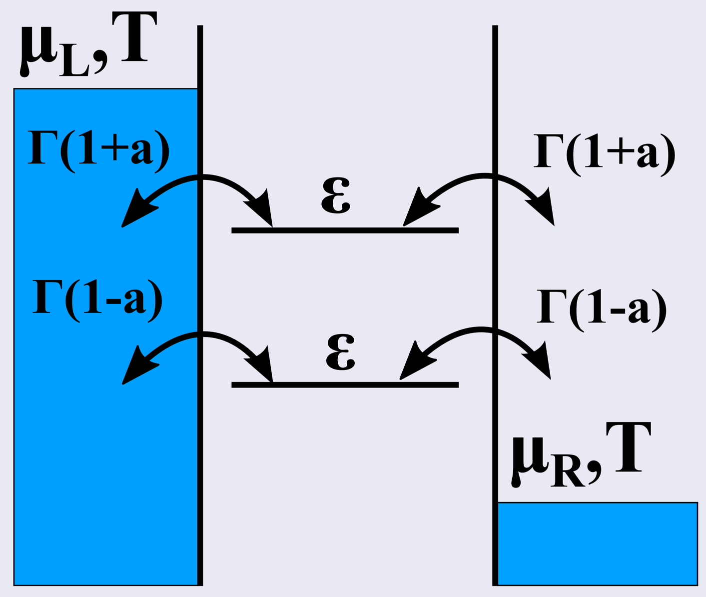
- **Fermionic and bosonic systems** – current cumulants (related to moments) calculated analytically using Levitov-Lesovik formula

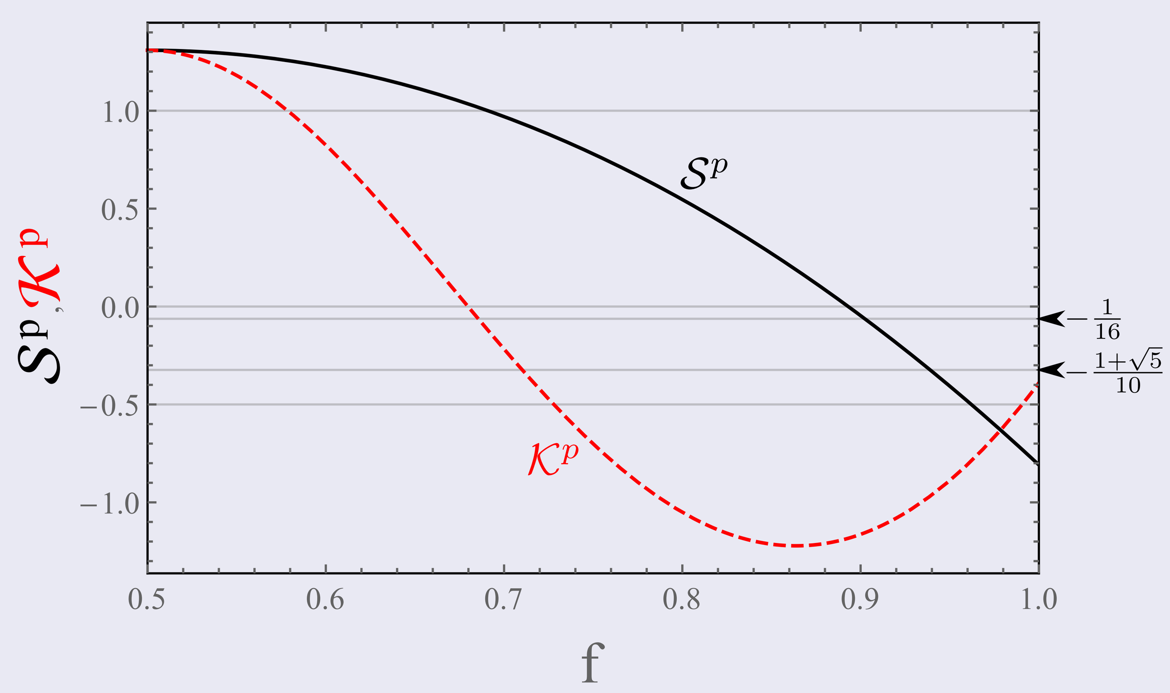
$$c^p_{\gamma\to\alpha,n} = \left[\frac{\partial^n}{\partial\lambda^n} \chi^p_{\alpha\gamma}(\lambda) \right]_{\lambda=0}$$

$$\chi^p_{\alpha\gamma}(\lambda) = \pm \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \ln \left\{ 1 \pm \mathcal{T}_{\alpha\gamma}(\omega) \left[\left(e^{\lambda} - 1\right) f_\gamma(\omega) g_\alpha(\omega) + \left(e^{-\lambda} - 1\right) f_\alpha(\omega) g_\gamma(\omega) \right] \right\}$$

- +/- – fermions/bosons, $f_\alpha(\omega)$ – Fermi/Bose-Einstein distribution, $g(\omega) = 1 - f(\omega)$
- $\langle j \rangle = c_1$, $\langle \Delta j^2 \rangle = c_2$, $\langle \Delta j^3 \rangle = c_3$, $\langle \Delta j^4 \rangle = c_4 + 3c_2^2$
- **Markovian systems** – numerical simulations of networks with different number of states

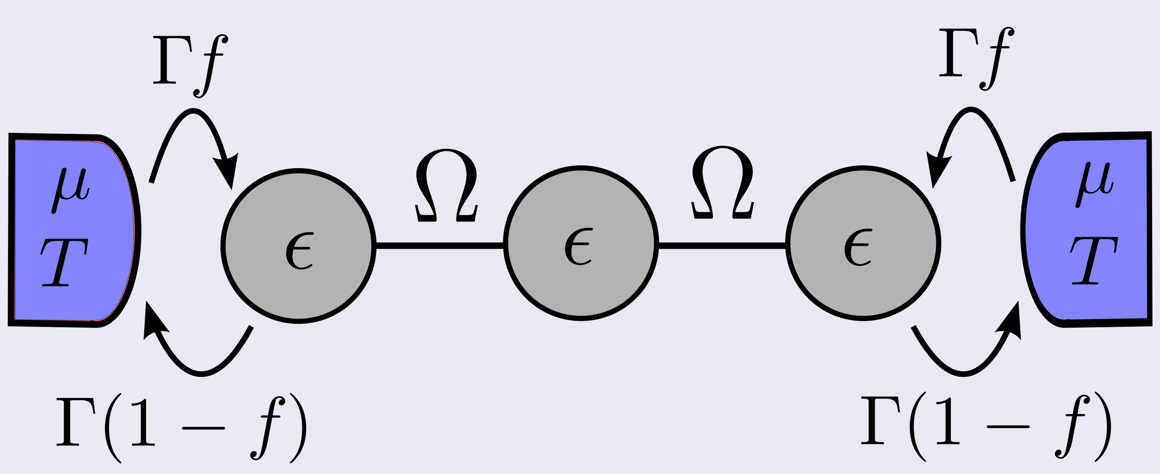
Counterexample: dynamical channel blockade



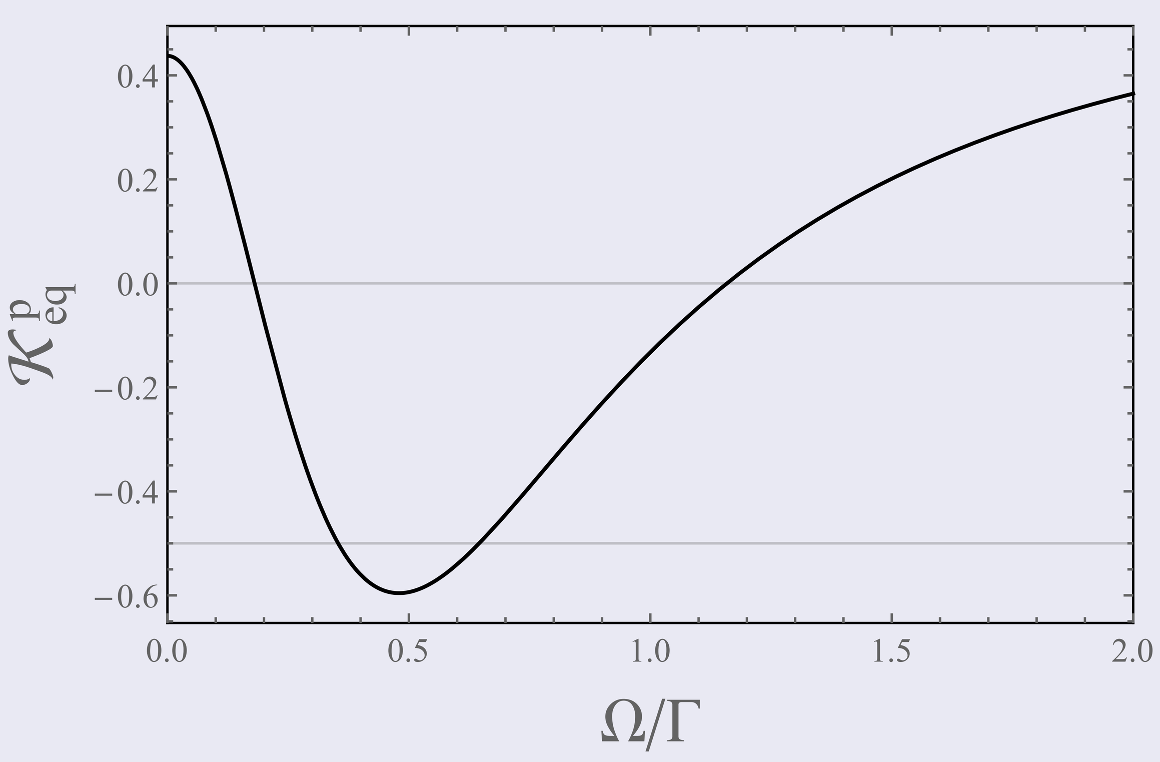


- Model: two level unequally coupled to electronic baths
- Strong Coulomb blockade – only one of the level can be occupied
- $\epsilon = 0$, $\mu_L = -\mu_R$, $f_L(\epsilon) = 1 - f_R(\epsilon) = f$, $a = 0.65$
- Breaking of bound 1. – presence of interactions
- Breaking of bound 3. – driving by voltage
- Breaking of bounds 4. – multicyclic Markovian network (different states $\longrightarrow$ different cycles)

Counterexample: triple quantum dot



- Model: triple quantum dot molecule attached to fermionic leads (classical jumps + internal coherent oscillations)
- Equilibrium: $f_L(\epsilon) = f_R(\epsilon) = 0.5$
- Breaking of bound 1. – presence of interactions
- Breaking of bound 5. – violation of classical Markovianity (due to unitary dynamics)



Acknowledgments

K. P. is supported by the National Science Centre, Poland, under the project Preludium 14 (No. 2017/27/N/ST3/01604).

References

[1] S. Saryal, H. Friedman, D. Segal, and B. K. Agarwalla, Phys. Rev. E **100**, 042101 (2019)