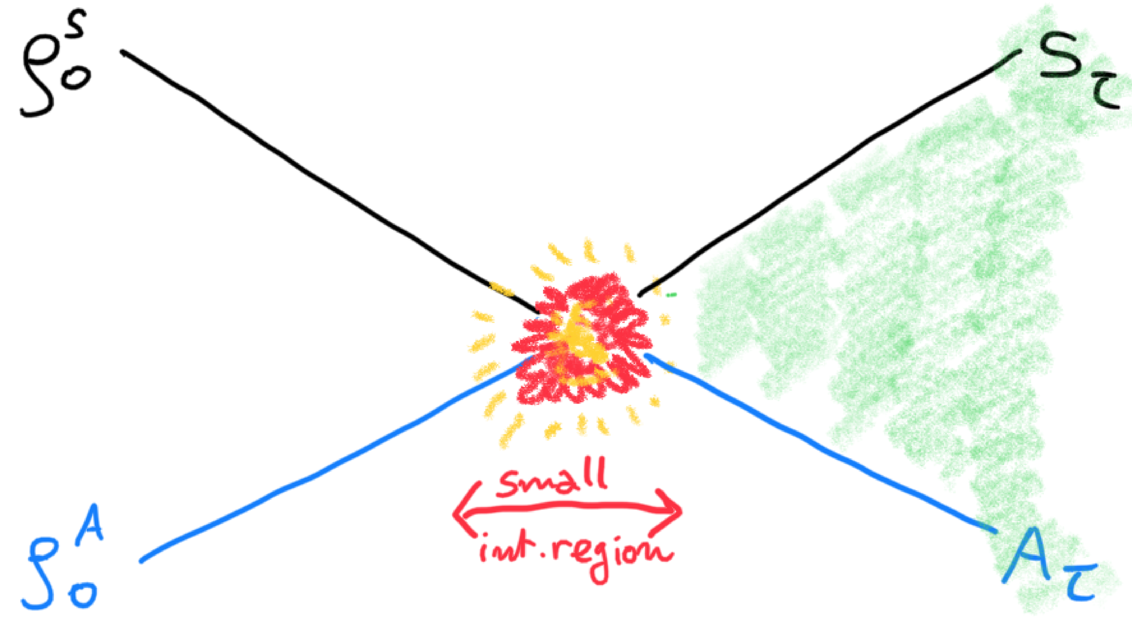
Thermodynamic Reverse Bounds

{ Francesco Buscemi } Dept. of Mathematical Informatics, Nagoya University

東海国立大学機構

1. Open quantum systems thermodynamics

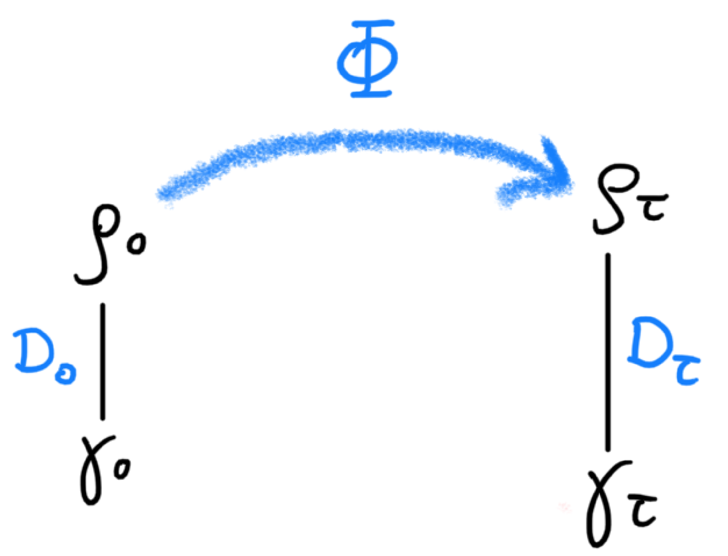
- system–ancilla initial factorization: $\rho_0^{SA} = \rho_0^S \otimes \rho_0^A$
- total Hamiltonian: $H^S(t) + H^A(t) + h^{SA}(t)$, for $0 \le t \le \tau$
- $\rho_\tau^S := \mathrm{Tr}_A \left\{ U_{0\to\tau}^{SA} \left(\rho_0^S \otimes \rho_0^A\right) \left(U_{0\to\tau}^{SA}\right)^\dagger \right\} =: \Phi(\rho_0^S)$
- system's average energy change: $\Delta E \approx \mathrm{Tr}\{\rho_\tau^S\, H^S(\tau)\} - \mathrm{Tr}\{\rho_0^S\, H^S(0)\}$



2. Energy change as an information divergence

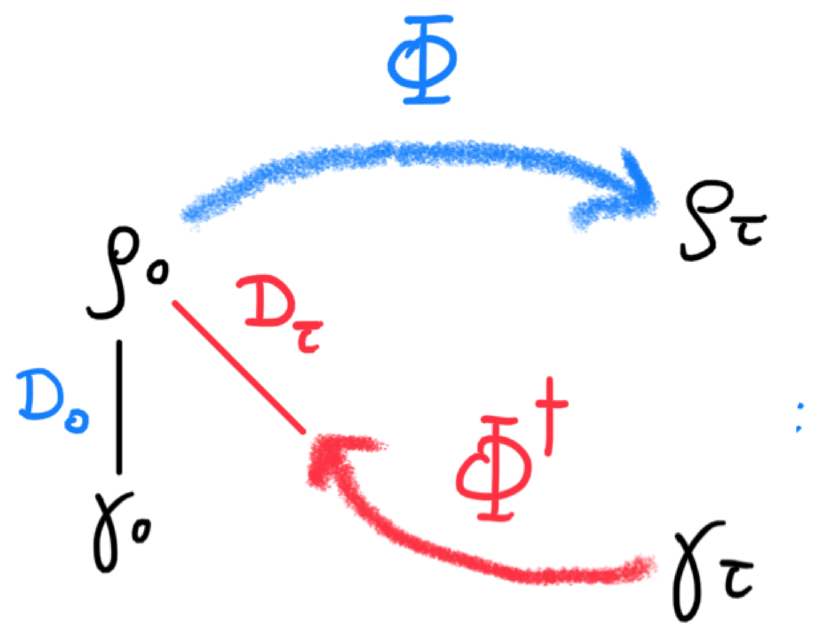
- Usual bound (γ is thermal state):

$$\beta(\Delta E - \Delta F) = \Delta D(\rho^S \| \gamma^S) + \Delta S \lesseqgtr \Delta D(\rho^S \| \gamma^S)$$



- "Reverse" bound:

$$\beta(\Delta E - \Delta F) \ge D(\rho_0^S \| \Phi^\dagger(\gamma_\tau^S)) - D(\rho_0^S \| \gamma_0^S)$$



3. Reverse bound and approximate recoverability

$$\Delta S = S(\Phi(\rho)) - S(\rho) = D(\rho \| \Phi^\dagger(\sigma)) - D(\Phi(\rho)\|\sigma) + \mathrm{Tr}[\rho\, \underbrace{(\ln \Phi^\dagger \sigma - \Phi^\dagger \ln \sigma)}_{[\ln, \Phi^\dagger] \ge 0}]$$

$$\ge D(\rho \| \Phi^\dagger(\sigma)) - D(\Phi(\rho)\|\sigma)$$

- assumption: $\Phi^\dagger$ a positive, unit-preserving linear map (Choi, 1974)
- for $\sigma = \Phi(\rho)$: $S(\Phi(\rho)) - S(\rho) \ge D(\rho \| \Phi^\dagger \circ \Phi(\rho))$, particularly meaningful for bistochastic channels

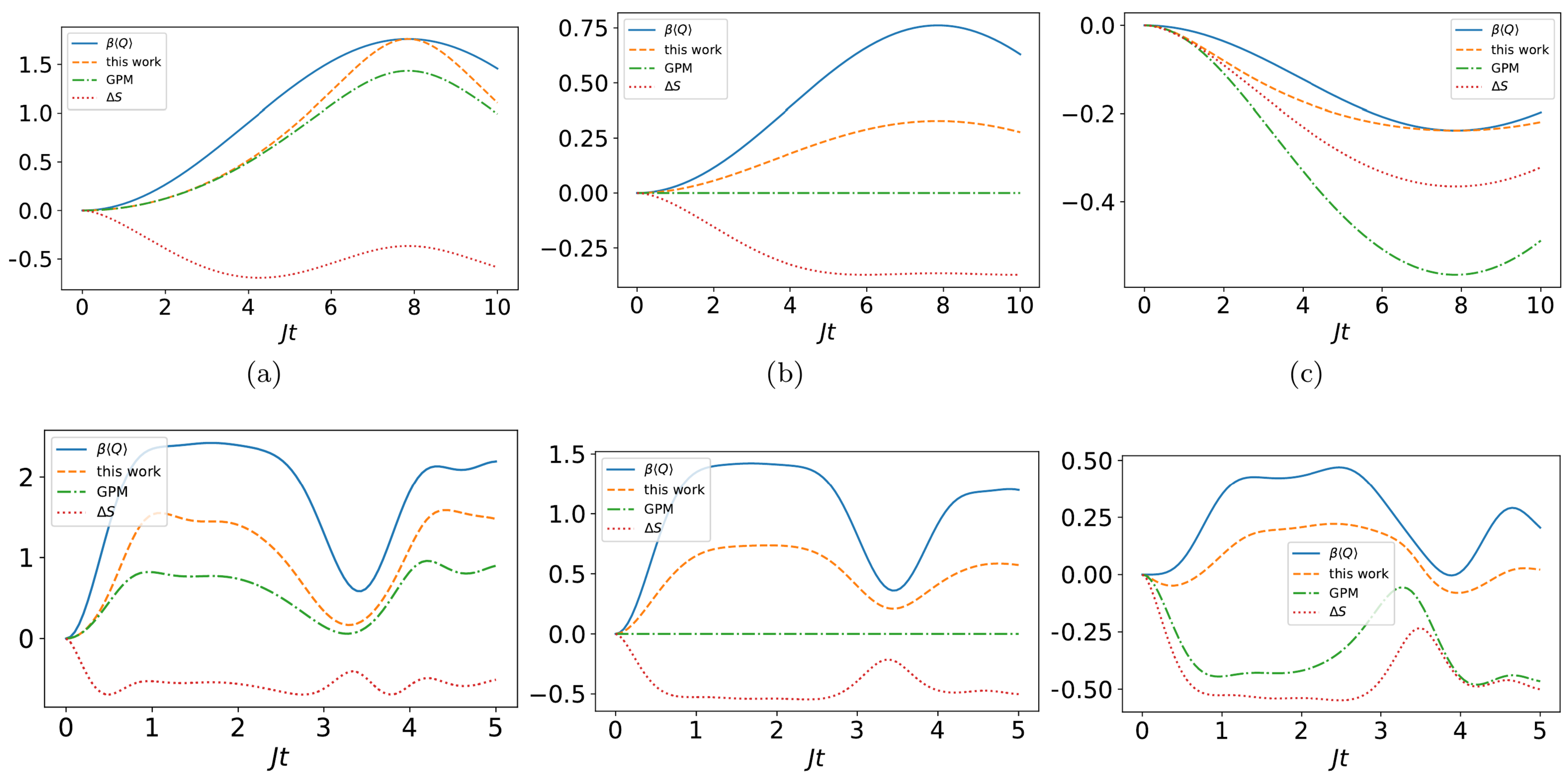
4. The "thermal pullback"

$$\beta(\Delta E - \Delta F) \ge D(\rho_0^S \| \Phi^\dagger(\gamma_\tau^S)) - D(\rho_0^S \| \gamma_0^S)$$
$$= D(\rho_0^S \| \tilde{\gamma}_0^S) - D(\rho_0^S \| \gamma_0^S) - \ln \mathrm{Tr}\{\Phi^\dagger(\gamma_\tau^S)\} \qquad \text{with } \tilde{\gamma}_0^S := \frac{\Phi^\dagger(\gamma_\tau^S)}{\mathrm{Tr}\{\Phi^\dagger(\gamma_\tau^S)\}}$$

- the value $\mathrm{Tr}\{\Phi^\dagger(\gamma_\tau^S)\}$ is called *efficacy*: it often appears in fluctuation relations (e.g., Albash&al 2013, Goold&al 2015)
- the pullback mapping $x \to \frac{\Phi^\dagger(x)}{\mathrm{Tr}\{\Phi^\dagger(x)\}}$ is CPTP but (in general) nonlinear

Application: erasure processes

$$\rho_0^S = \gamma_0^S,\ \Delta F = 0,\ \Delta E = \langle Q \rangle \text{ [see Ref. [2]]}$$



(a) (b) (c)

(d) (e) (f)

The continuous curve is $\beta\langle Q \rangle$, the dashed curve refers to the bound in [2], the dashed-dotted curve refers to [Goold, Paternostro, Modi: Phys. Rev. Lett. 114, 060602, 2015], and the dotted curve is ΔS.

References

[1] F. Buscemi. Approximate reversibility in the context of entropy gain, information gain, and complete positivity. PRA, 93, 2016.

[2] F. Buscemi, D. Fujiwara, N. Mitsui, and M. Rotondo. Thermodynamic reverse bounds for general open quantum processes. PRA, 102(3), 2020.

[3] F. Buscemi and V. Scarani. Fluctuation theorems from Bayesian retrodiction. PRE, 103(5), 2021.

Further reading: retrodiction

The thermal pullback is intimately related with the concept of retrodiction in extended logic, derived either from Jeffrey's probability kinematics or from Pearl's method of virtual evidence. See [3] also for compelling connections between fluctuation relations, the second law of thermodynamics, and Bayes' theorem, thought of as a condition for consistent reasoning.