Work extraction from unknown quantum sources

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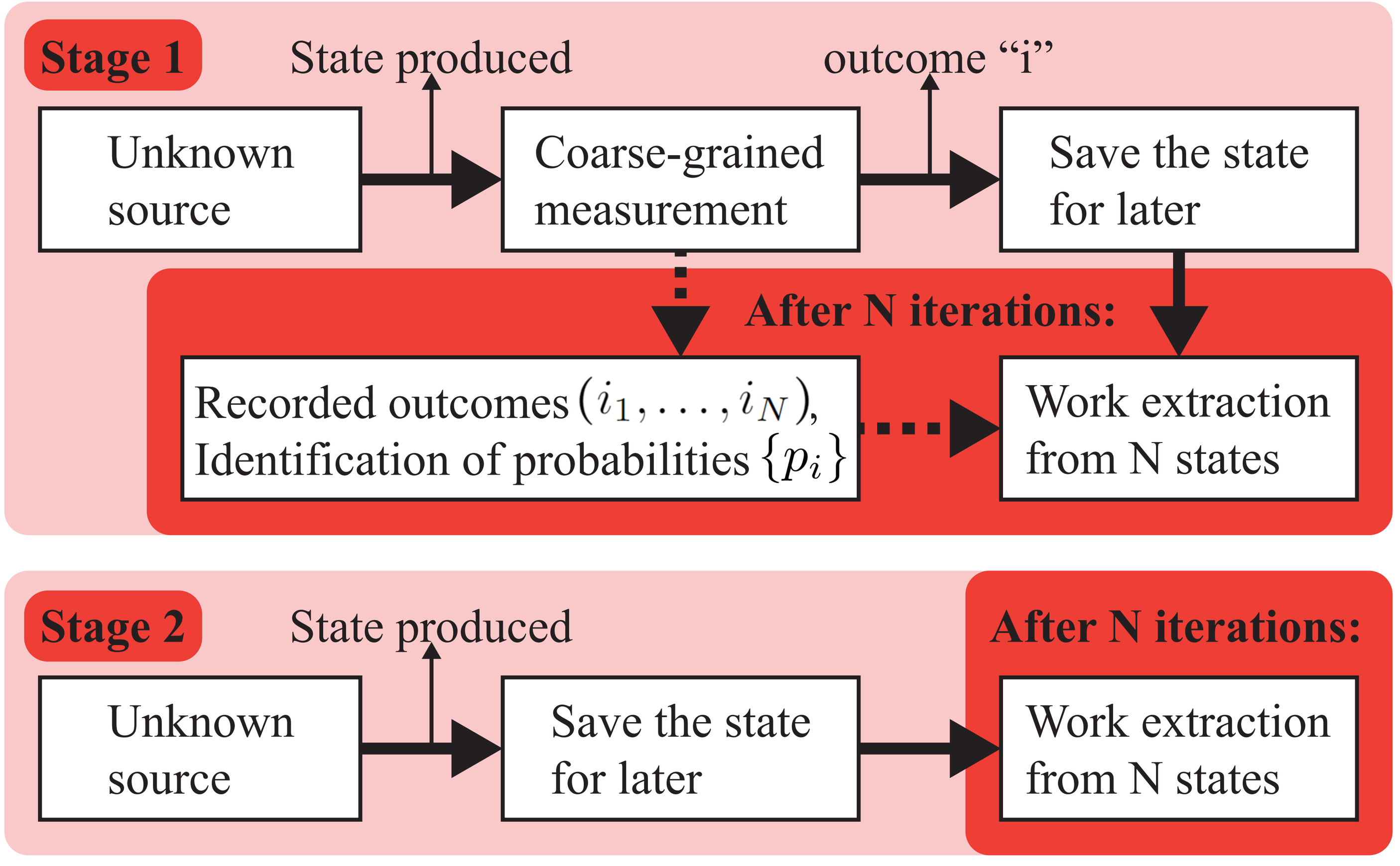
Problem

- We assume completely unknown source of quantum states.
- We can measure and characterize it by only a single type of coarse-grained measurement.
- The allowed extraction is assumed to be a type of unitary, which does not allow manipulation of inner structure of each macrostate. Modelled as:

$$U \bigoplus_i U_i,$$

where U_i are random.

- Given these assumptions, can we still extract work from it?
- How much can we extract?



Measurement = Coarse-graining

- Coarse-graining = partition of the Hilbert space $\mathcal{H} = \mathcal{H}_1 \oplus \cdots \oplus \mathcal{H}_N$
- Each observable defines a coarse-graining $\hat{A} = \sum_i a_i \hat{P}_i, \quad \hat{P}_i \leftrightarrow \mathcal{H}_i$ defines macrostate.

Results

Yes, we can extract work.

In Stage 1, it is given by [Boltzmann ergotropy]

$$W_{\mathcal{C}}^{B\infty}(\rho) := \lim_{N\to\infty} \frac{W_{\mathcal{C}}^{B}(\rho^{\otimes N})}{N} = \mathrm{tr}[H(\rho - \rho_\beta)],$$

where temperature is implicitly defined by

$$S_{\mathrm{vN}}(\rho_\beta) = \sum_i p_i \ln V_i =: S_{\mathcal{C}}^{B}$$

(Mean Boltzmann entropy on the RHS)

Extracting from

$$\rho^N \approx \bigotimes_i \rho_i^{\otimes p_i N}$$

In Stage 2, it is given by [Observational ergotropy]

$$W_{\mathcal{C}}^{\infty}(\rho) := \lim_{N\to\infty} \frac{W_{\mathcal{C}}(\rho^{\otimes N})}{N} = \mathrm{tr}[H(\rho - \rho_{\beta'})]$$

where temperature is implicitly defined by

$$S_{\mathrm{vN}}(\rho_{\beta'}) = -\sum_i p_i \ln p_i + \sum_i p_i \ln V_i = S_{\mathcal{C}}^{Sh} + S_{\mathcal{C}}^{B} = S_{\mathcal{C}}$$

(Observational entropy on the RHS)

Extracting from

$$\rho_{\mathrm{cg}}^{\otimes N} = \Big(\sum_i p_i P_i / V_i\Big)^{\otimes N}$$

Observational entropy

Measures observers uncertainty about the system provided they can only measure certain macroscopic measurements (those are given by the coarse-graining)

$$S_{\mathcal{C}} \equiv S_{\mathcal{C}_1,\dots,\mathcal{C}_n} \equiv -\sum_i p_i \ln p_i + \sum_i p_i \ln V_i$$

$$p_i = \mathrm{tr}\big[\hat{P}_i \hat{\rho}\big]$$

$$V_i = \mathrm{tr}\big[\hat{P}_i \hat{\rho}\big]$$

uncertainty about into which macrostate the system belongs to
Shannon entropy = uncertainty from receiving different answers

Mean uncertainty after performing a measurement
Mean Boltzmann entropy = mean logarithm of No. of microstates

Example: Thermalization of Bose-Hubbard gas

1 - dimensional gas evolving with modified Bose-Hubbard Hamiltonian (to make it non-integrable), with Nearest Neighbor and Next Nearest Neighbor hopping.

$$H^{(k:l)} = \sum_{i=k}^{l} -t f_i^\dagger f_{i+1} - t' f_i^\dagger f_{i+2} + h.c. + V n_i n_{i+1} + V' n_i n_{i+2}$$

- Observational erogtropy starts large, and it is possible to exctract work.
- As the system thermalizes however, observational entropy grows, and observational ergotropy decreases.
- When the lower bound on ergotroopy passes through zero, the experimenter cannot be sure to extract positive work.
- Thus, the source gets progressively worse with time, one shouldn't wait for the system to thermalize!

Note: observational ergotropy depends on the initial energy, which is unknown, but can be estimated: that's why we have the shaded red region as the estimate.