Typical nonequilibrium steady current due to strong prethermalization

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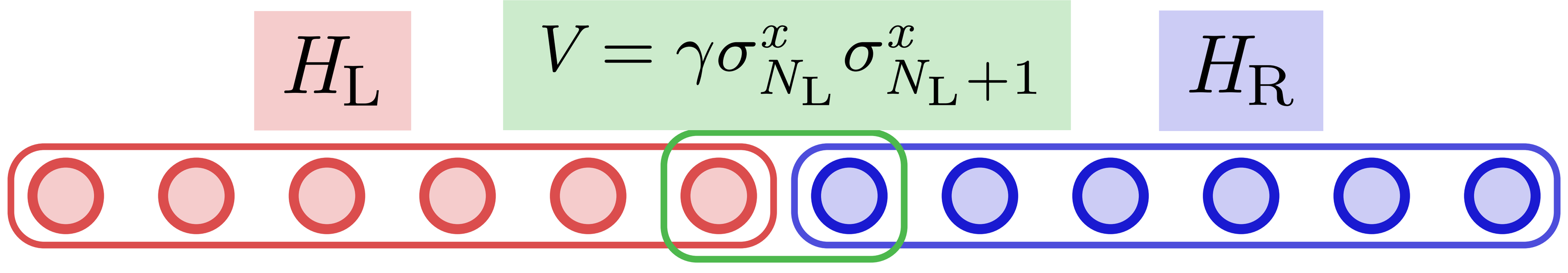
Introduction

Great insights on equilibration and thermalization of isolated quantum systems is given by pure state quantum statistical mechanics. Particularly important are the concepts of typicality and strong thermalization. Here we want to study whether pure state statistical mechanics can give insights on the emergence of non-equilibrium steady states and whether the concepts of typicality and strong thermalization can apply in these scenarios.

Model



Two nonintegrable spin chains are considered as baths $H = H_{\rm L} + H_{\rm R} + V$,e.g., the Hamiltonian of the left bath is given by

$$H_{\rm L} = \sum_{n=1}^{N_{\rm L}-1} J_{zz}\sigma_n^z\sigma_{n+1}^z + J_{yz}\sigma_n^y\sigma_{n+1}^z + \sum_{n=1}^{N_{\rm L}} h_x\sigma_n^x + h_z\sigma_n^z.$$

- $V = \gamma B^{\rm L} \bigotimes B^{\rm R}$ where $B^{\rm L} = \sigma^x_{N_{\rm L}}$ and $B^{\rm R} = \sigma^x_{N_{\rm L}+1}$
- $J_{yz} = J_{zz}$, $h_x = -1.05J_{zz}$, $h_z = 0.5J_{zz} \Longrightarrow$ Gaussian Unitary Ensemble.

Energy current

Energy current operator with respect to the left bath as

$$I^{\rm L} = \frac{dH_{\rm L}}{dt} = -{\rm i}[V, H_{\rm L}] = \dot{B}^{\rm L} \otimes B^{\rm R}$$

where we have defined $\dot{B}^{\rm L} \equiv -{\rm i}\gamma\left[B^{\rm L}, H_{\rm L}\right]$.

Exact current: When performing a exact calculation considering the full Hamiltonian, the current operator $I_{\rm L}$ is given by

$$I^{\rm L} = -2\gamma(J_{zz}\sigma^z_{N_{\rm L}-1}\sigma^y_{N_{\rm L}}\sigma^x_{N_{\rm L}+1} + J_{yz}\sigma^y_{N_{\rm L}-1}\sigma^y_{N_{\rm L}}\sigma^x_{N_{\rm L}+1} + h_z\sigma^y_{N_{\rm L}}\sigma^x_{N_{\rm L}+1}).$$

Perturbative current: Similar to the weak coupling master equations formalism, by treating coupling term V as a perturbation, one can find a perturbative current expression

$$\mathcal{I}(t) = \dot{\mathcal{B}}^{\rm L}(t)\mathcal{B}^{\rm R}(t) - {\rm i}\int_0^t d\tau\left[\overrightarrow{\mathcal{C}}_{\rm L}(t,\tau)\mathcal{C}_{\rm R}(t,\tau) - \overleftarrow{\mathcal{C}}_{\rm L}(\tau,t)\mathcal{C}_{\rm R}(\tau,t)\right],$$

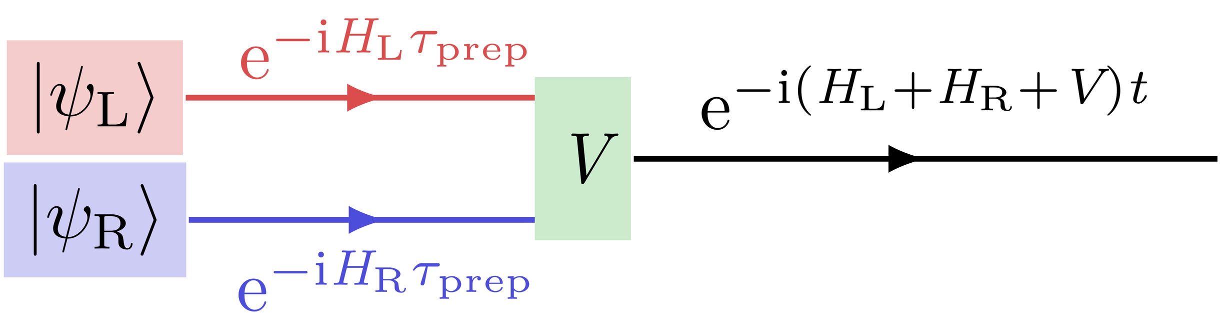
where $\dot{\mathcal{B}}^{\rm L}(t) = {\rm tr}_{\rm L}\left[\rho_{\rm L}\dot{B}^{\rm L}(t)\right]$, $\mathcal{B}^{\rm R}(t) = {\rm tr}_{\rm R}\left[\rho_{\rm R}B^{\rm R}(t)\right]$ and we have defined the two-time correlation functions $\overrightarrow{\mathcal{C}}_{\rm L}(t,\tau) = {\rm tr}_{\rm L}\left[\rho_{\rm L}\dot{B}^{\rm L}(t)B^{\rm L}(\tau)\right]$, $\overleftarrow{\mathcal{C}}_{\rm L}(t,\tau) = {\rm tr}_{\rm L}\left[\rho_{\rm L}B^{\rm L}(\tau)\dot{B}^{\rm L}(t)\right]$, and $\mathcal{C}_{\rm R}(t,\tau) = {\rm tr}_{\rm R}\left[\rho_{\rm R}B^{\rm R}(t)B^{\rm R}\right]$.

Initial state and preparation protocol

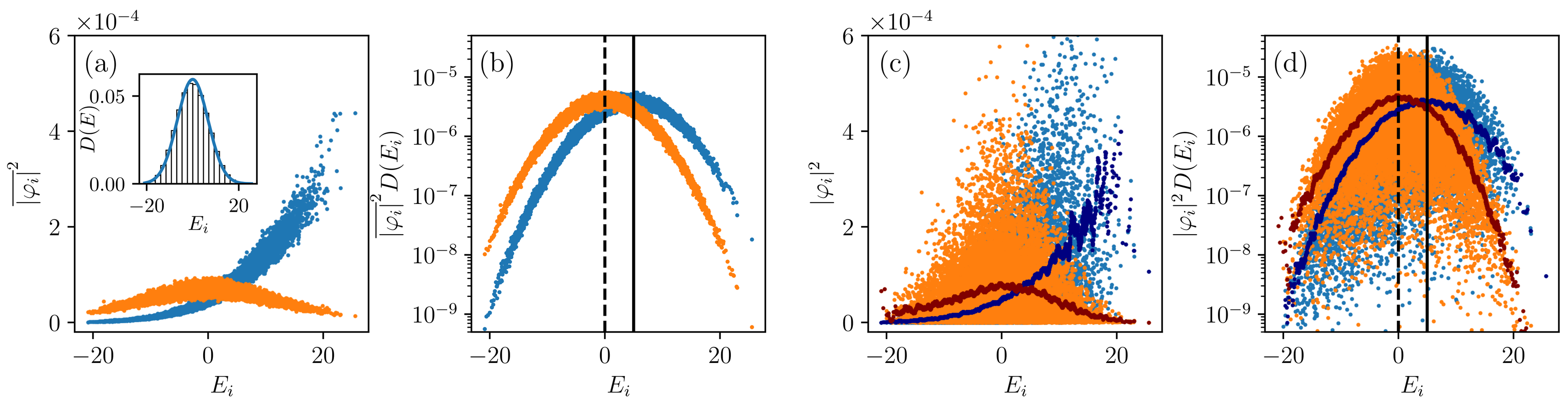
Energy constraint: $|\langle\psi_\alpha|H_\alpha|\psi_\alpha\rangle - \mathcal{E}_\alpha| < \eta_\alpha$
where $\alpha = {\rm L}, {\rm R}$ refer to the left or right environment and η_α are small real numbers.

Random product states $|\psi_\alpha\rangle = \bigotimes_n^{N_\alpha}(u_n|\uparrow\rangle_n + v_n|\downarrow\rangle_n)$
where $|u_n|^2 + |v_n|^2 = 1$ and u_n and v_n are sampled, or optimized, such that they satisfy the above constraint.

Preparation protocol:



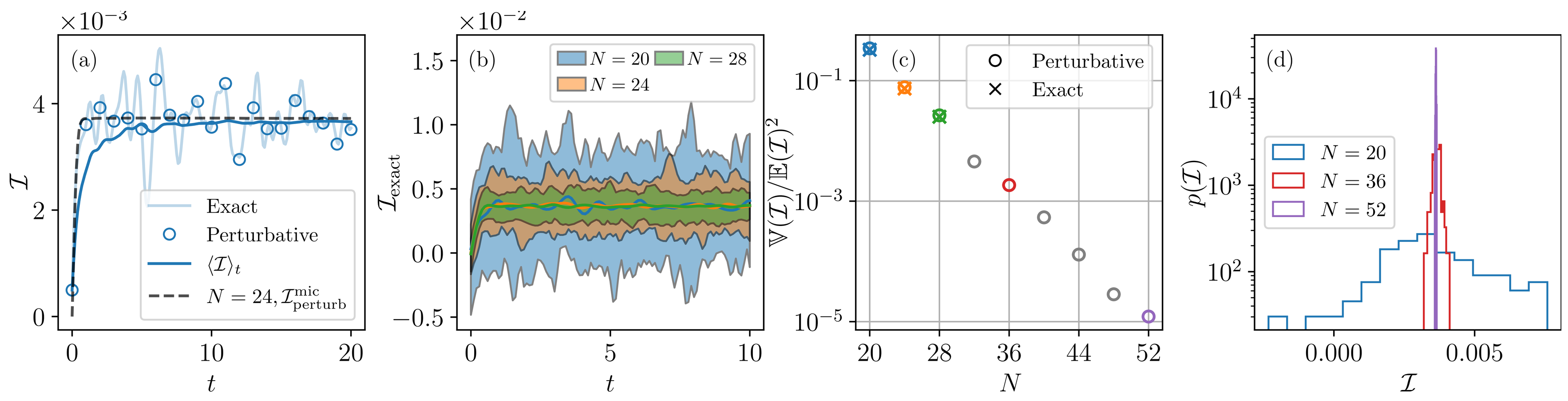
Typicality in the environment



(a, b) Average over 100 states (c, d) A single realization

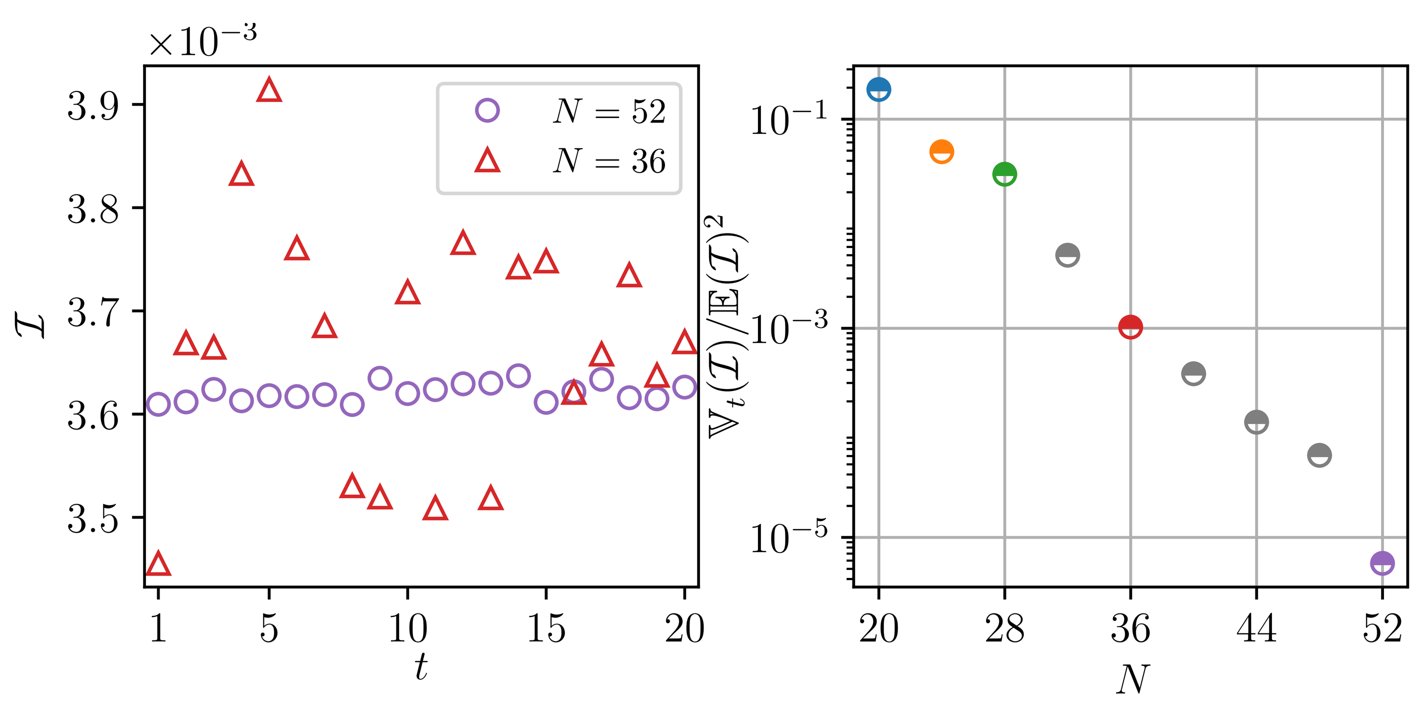
(a) Initial state profiles in the eigenbasis of the bath, e.g., $H_{\rm L}$
(b) Moving averages of a single realization in navy and maroon are consistent with the averages.
(c) Target energies $\mathcal{E}_{\rm L} = \mathcal{N}_{\rm L}\varepsilon_{\rm L}$ are indicated by the black solid ($\varepsilon_{\rm L} = 5/12$) and dashed lines ($\varepsilon_{\rm L} = 0$).
(d) $D(E)$: Many-body density of states

Typicality for nonequilibrium currents



(a) Benchmarking the exact and perturbative methods. Comparing time-averged current and microcanonical current.
(b) Exact time evolution for 100 samples for $N = 20, 24, 28$.
(c) **Exponential decay of the variance of currents at $t = 5$**
(d) Distribution of the currents at $t = 5$ for $N = 20, 36, 52$

Steady current and strong prethermalization



(a) Current dynamics for $N = 36$ and $N = 52$.
(b) **Exponential decay of the variance over time for currents for a single realization**

Conclusions and Discussions

Steady current is a manifestation of **prethermalization** in the strong sense, i.e., the current values will stay close to the ensemble averaged one for most of the time. The fluctuation of current decays with environment size exponentially both in terms of sample variance and variance over time.

[1] X. Xu, C. Guo, D. Poletti, (Submitting)
[2] X. Xu, C. Guo, D. Poletti, Phys. Rev. A **105**, L040203 (2022)

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