Maximum power heat engines and refrigerators in the fast-driving regime

Vasco Cavina [1] Paolo A. Erdman [2] Paolo Abiuso [3] Leonardo Tolomeo [4] Vittorio Giovannetti [5]

[1]Complex Systems and Statistical Mechanics, Physics and Materials Science,University of Luxembourg, L-1511 Luxembourg, Luxembourg [2]Freie Universität Berlin, Department of Mathematics and Computer Science, Arnimallee 6, 14195 Berlin, Germany
[3]ICFO – Institut de Ciències Fotòniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain [4]Mathematical Institute, Hausdorff Center for Mathematics, Universität Bonn, 456-321 Bonn, Germany
[5]NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, I-56126 Pisa, Italy

Goal: optimization of thermal machines

Within the assumption of fast modulation of the driving parameters, we derive the optimal cycle that universally maximizes the extracted power of heat engines, the cooling power of refrigerators, and in general any linear combination of the heat currents. In our general setting, the system is in contact with N-baths, with associated temperatures β_α and dissipators $\mathcal{D}_\alpha$ where $\alpha = 1, ..N$, the associated GKSL master equation being

$$\partial_t \rho(t) = \mathcal{L}_{\vec{u}(t)}\left[\rho(t)\right] \equiv -\frac{i}{\hbar}\left[H_{\vec{u}(t)}, \rho(t)\right] + \sum_{\alpha=1}^{N} \mathcal{D}_{\alpha,\vec{u}(t)}\left[\rho(t)\right], \quad (1)$$

where we also assumed the dissipators and the Hamiltonian to depend on a vector of M external controls $\vec{u}(t)$ (see also fig. 1).

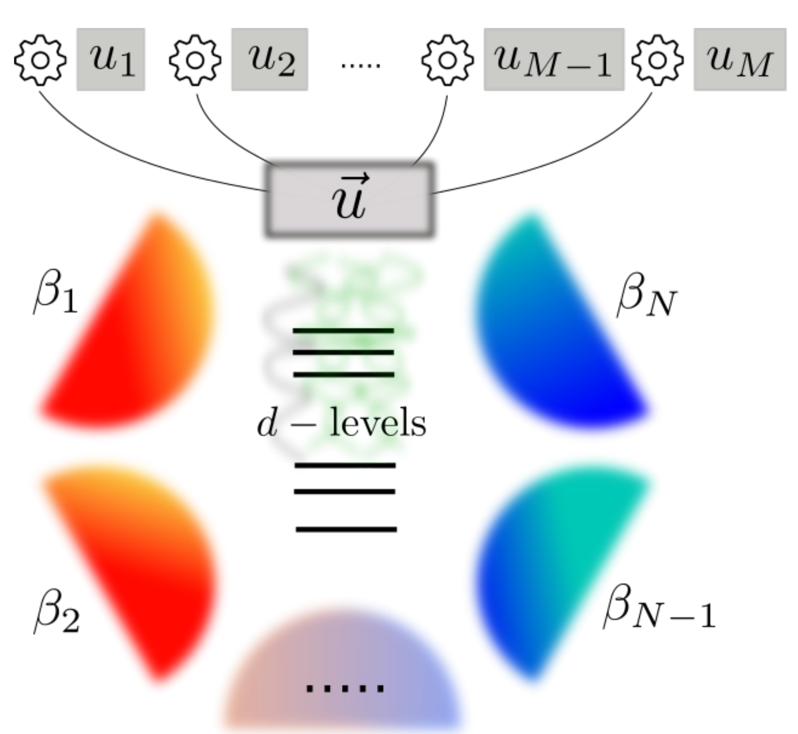

Figure 1. An arbitrary d-level system, controlled by M parameters $\vec{u}(t)$, is coupled to N baths.

Introducing the heat current flowing in a single bath as

$$J_\alpha(t) \equiv \left[H_{\vec{u}(t)} \mathcal{D}_{\alpha,\vec{u}(t)}\left[\rho(t)\right]\right], \quad (2)$$

we can write a generalized power (GAP) as a time averaged linear combinations of currents

$$P_{\mathbf{c}}[\vec{u}] \equiv \frac{1}{T}\sum_{\alpha=1}^{N}\int_0^T c_\alpha J_\alpha(t)\, dt, \quad (3)$$

whith T being the period, and c_α the weights that determine the specific GAP we want to optimize (for instance, the power is obtained with $c_\alpha = 1$).

The fast driving regime

Minimizing (3) turns out to be simpler if T is much shorter than the typical baths-induced relaxation times. In this case, after a certain amount of time, we assume the system to converge to a time-independent, out-of-equilibrium state $\rho^{(0)}_{[\vec{u}]}$. Eq. (3) simplifies to

$$P_{\mathbf{c}}[\vec{u}] = \frac{1}{T}\int_0^T \left[H_{\vec{u}(t)} \sum_\alpha c_\alpha \mathcal{D}_{\alpha,\vec{u}(t)}\left[\rho^{(0)}_{[\vec{u}]}\right]\right] dt. \quad (4)$$

The cut and choose idea

Instead of a direct constrained functional optimization of the GAP [see Eq. (4)] with respect to $\vec{u}(t)$, we will employ an iterative procedure. The main idea of the proof (that we call "cut and choose") is the following: given any assigned periodic protocol $\vec{u}$, we prove that it is possible to "cut away" parts of it to build a new, shorter, cycle which delivers a higher or equal GAP than the starting one.

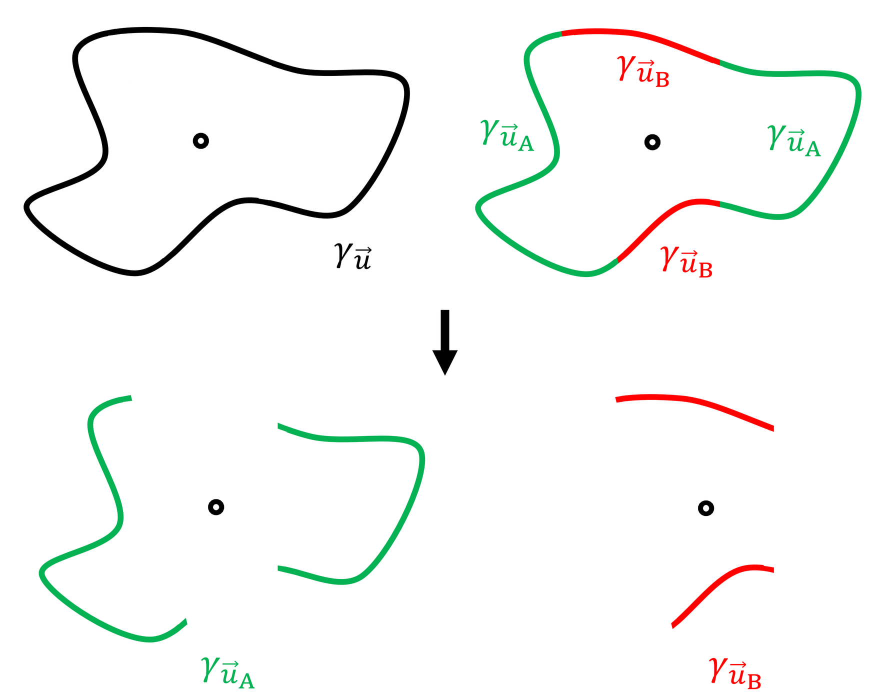

Figure 2. An example of how a path in the parameter space can be cut in two parts, labelled with A and B. One of the two delivers a greater GAP than the original path.

More precisely, we can always divide a path $\gamma_{[\vec{u}]}$ in two paths $\gamma_{[\vec{u}_A]}$ and $\gamma_{[\vec{u}_B]}$ such that the GAP satisfies

$$P_{\mathbf{c}}[\vec{u}] = \frac{T_A P_{\mathbf{c}}[\vec{u}_A] + T_B P_{\mathbf{c}}[\vec{u}_B]}{T_A + T_B}, \quad (5)$$

that allows to establish that one of the two subprotocols achieves a bigger GAP. The existence of such partitions can be proved using the implicit function theorem [1]. By reiterating this process over and over, we end up with the optimal protocol, the "generalized Otto cycle".

The universal optimal protocols: generalized Otto cycles

. An optimal protocol is one in which the "cut and choose" procedure cannot be further iterated. We formally proved in [1] that the protocols with such a property are "infinitesimal", that is, composed by little steps in which the integrand of (3) is approximately constant, that is

$$P_{\mathbf{c}}[\{\vec{u}_i, \mu_i\}] = \sum_{j=1}^{L} \mu_j \left[H_{\vec{u}_j} \sum_\alpha c_\alpha \mathcal{D}_{\alpha,\vec{u}_j}\left[\rho^{(0)}_{[\{\vec{u}_i,\mu_i\}]}\right]\right], \quad (6)$$

where $\mu_i = d\tau_i/d\tau$ represents the percentage of the total protocol time spent at each point $\vec{u}_i$. In addition, we proved that the dimension of the Hilbert space d bounds the number of steps L of the generalized Otto cycle, that is $L \leq d$.

Applications

Fast driving turns out to be the optimal regime for power and cooling power optimization in a variety of frameworks [2, 3, 4, 5]. In all these frameworks, our results can be used to greatly simplify the computational complexity of the optimization procedure. As an example, in [1] we considered a simple relaxation case in which all the elements of the density matrix converge to a Gibbs fixed point with the same rate. Using generalized Otto cycles, we can analytically solve the power optimization for this simple model. In particular, we can compare the maximum power in a system of N interacting qubits (a case in which we have the full control fo the many body hamiltonian) with the non interacting case in which we control only the local Hamiltonian of the qubits. In the small N regime, we see that there is a many body advantage in the interacting case, as shown in figure (3).

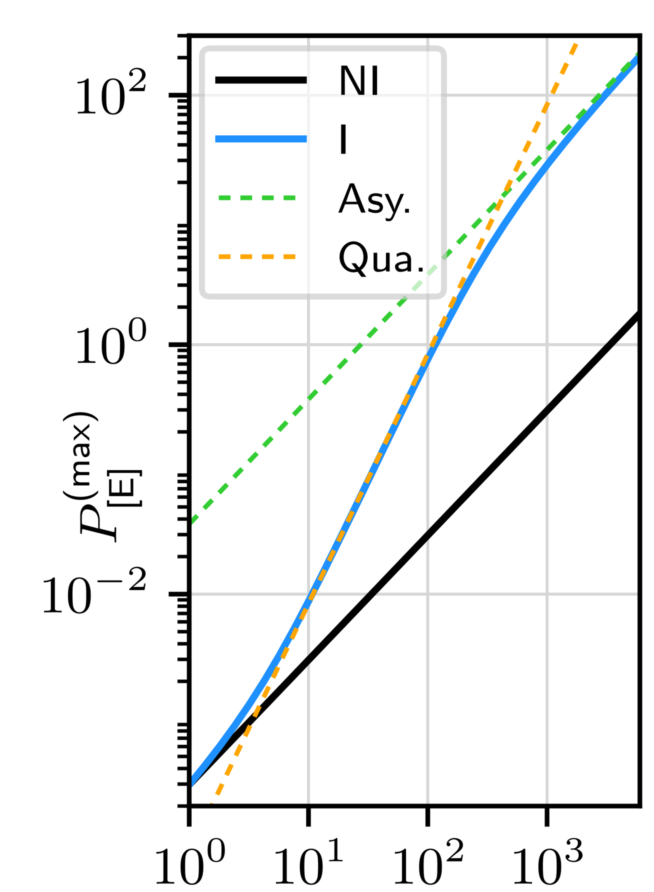

Figure 3. Maximum power for the optimal protocol, measured in units of a reference temperature and rate [1], as a function of the number of qubits n displayed in a log-log plot. The black curve corresponds to the non-interacting case, while the blue curve to the interacting case. The dashed green curve is the analytic asymptotic value we find for $n \to \infty$ while the dashed orange line is a reference quadratic function.

References

[1] Vasco Cavina, Paolo A Erdman, Paolo Abiuso, Leonardo Tolomeo, and Vittorio Giovannetti. Maximum-power heat engines and refrigerators in the fast-driving regime. *Physical Review A*, 104(3):032226, 2021.

[2] Vasco Cavina, Andrea Mari, Alberto Carlini, and Vittorio Giovannetti. Optimal thermodynamic control in open quantum systems. *Physical Review A*, 98(1):012139, 2018.

[3] Paolo Andrea Erdman, Vasco Cavina, Rosario Fazio, Fabio Taddei, and Vittorio Giovannetti. Maximum power and corresponding efficiency for two-level heat engines and refrigerators: optimality of fast cycles. *New Journal of Physics*, 21(10):103049, 2019.

[4] Eitan Geva and Ronnie Kosloff. A quantum-mechanical heat engine operating in finite time. a model consisting of spin-1/2 systems as the working fluid. *The Journal of chemical physics*, 96(4):3054–3067, 1992.

[5] Tim Schmiedl and Udo Seifert. Efficiency at maximum power: An analytically solvable model for stochastic heat engines. *EPL (Europhysics Letters)*, 81(2):20003, 2007.

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vasco.cavina@uni.lu