Quantum homogenization in non-Markovian collisional model

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Introduction

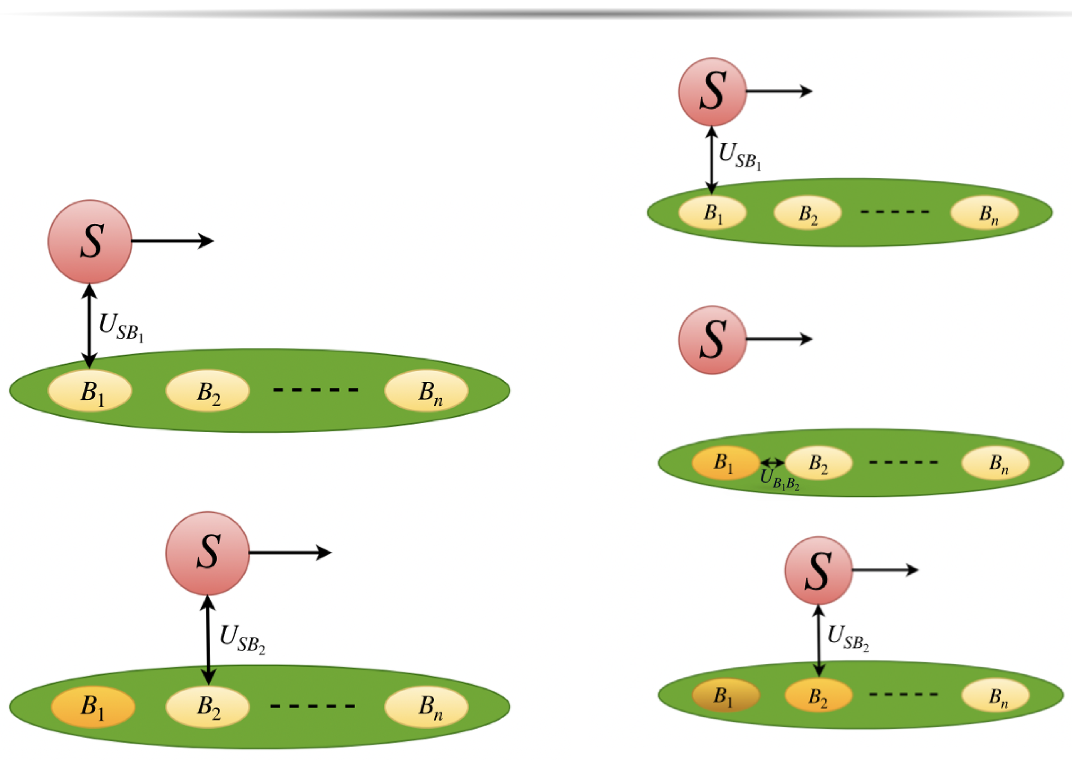
- Thermalization is one of the most fundamental problems of non-equilibrium thermodynamics.
- In the setting of Markovian Collisional Model (CM), thermalization problem was addressed earlier through a more general process, called Quantum Homogenization [1, 2].
- Quantum Homogenization is a process where the system state approaches the identically prepared state of bath unit in the asymptotic limit.

Objectives

- Does homogenization occur in Non-Markovian CM?
- Is it universal?

- In particular, Ziman et al. [1] showed that for a qubit system interacting with qubit ancillas, partial swap (PSWAP) is the unique system-ancilla unitary, for which homogenization is achieved regardless of the initial states of system and ancillas.

Collisional Model



- The most basic ingredients of CMs:
  (i) System of interest,
  (ii) Bath (or environment), made of identically prepared sub-units (ancillas)
  (iii) The system interacts sequentially with one ancilla at a time via a unitary.
- Such simplicity makes the CMs advantageous to study Non-Markovian dynamics by some modifications in the basic model outlined above:
  (a) by introducing ancilla-ancilla interaction,
  (b) using initially correlated bath,
  (c) through a composite collisional model, etc.

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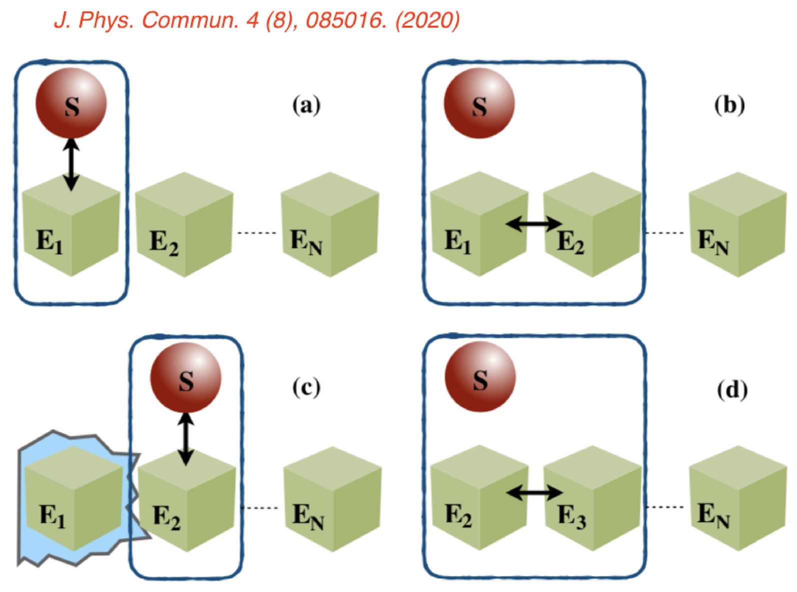


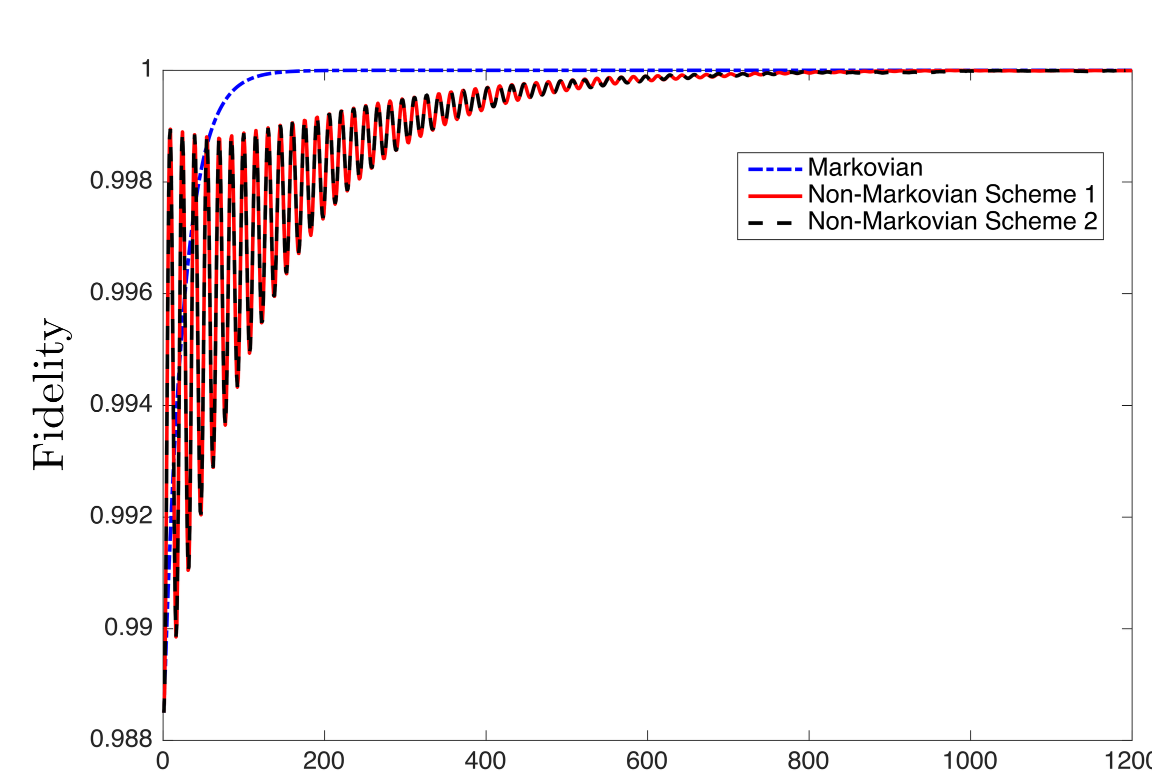
Framework

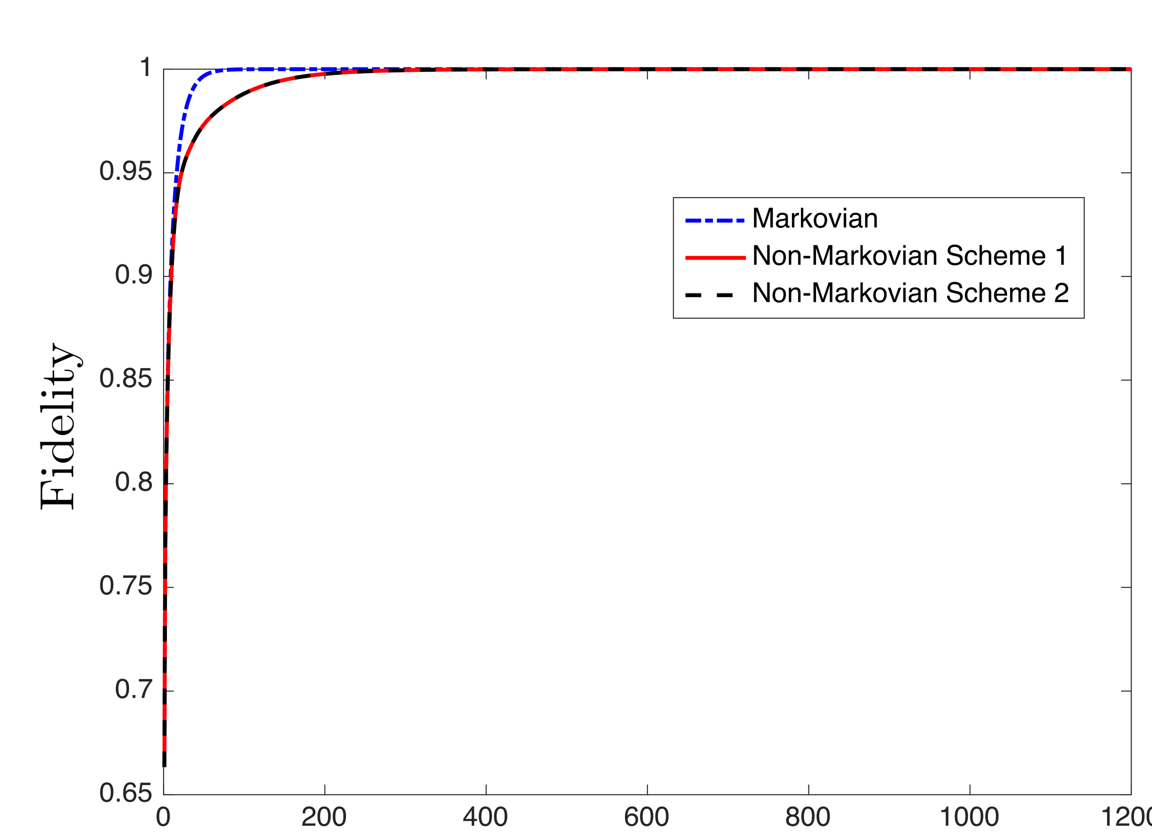
1. Markovian Collisional Model

- Arbitrary initial state of system and ancilla are:
$$\rho_0^S = \frac{1}{2}(\mathbb{1} + \vec{k}^{(0)}.\vec{\sigma})$$
$$\eta = \frac{1}{2}(\mathbb{1} + \vec{l}.\vec{\sigma})$$
- System-ancilla unitary:
$$U_{SB_n}(\alpha) = (\cos\alpha)\mathbb{1}_{4\times4} + i(\sin\alpha)S_{4\times4}$$
- Our goal: $\lim_{n\to\infty}\rho_n^S = \eta$
- Iff conditions for homogenization:
$$\mathrm{Tr}_S\{U(\rho\otimes\rho)U^\dagger\} = \rho \text{ and } \mathrm{Tr}_B\{U(\rho\otimes\rho)U^\dagger\} = \rho$$
- Ratio test helps to achieve the goal:
$$\vec{k}^{(n)} \longrightarrow \vec{l}, \quad \text{and} \quad \vec{l}_n^{(1)} \longrightarrow \vec{l}$$

2. Non-Markovian Collisional Model for PSWAP

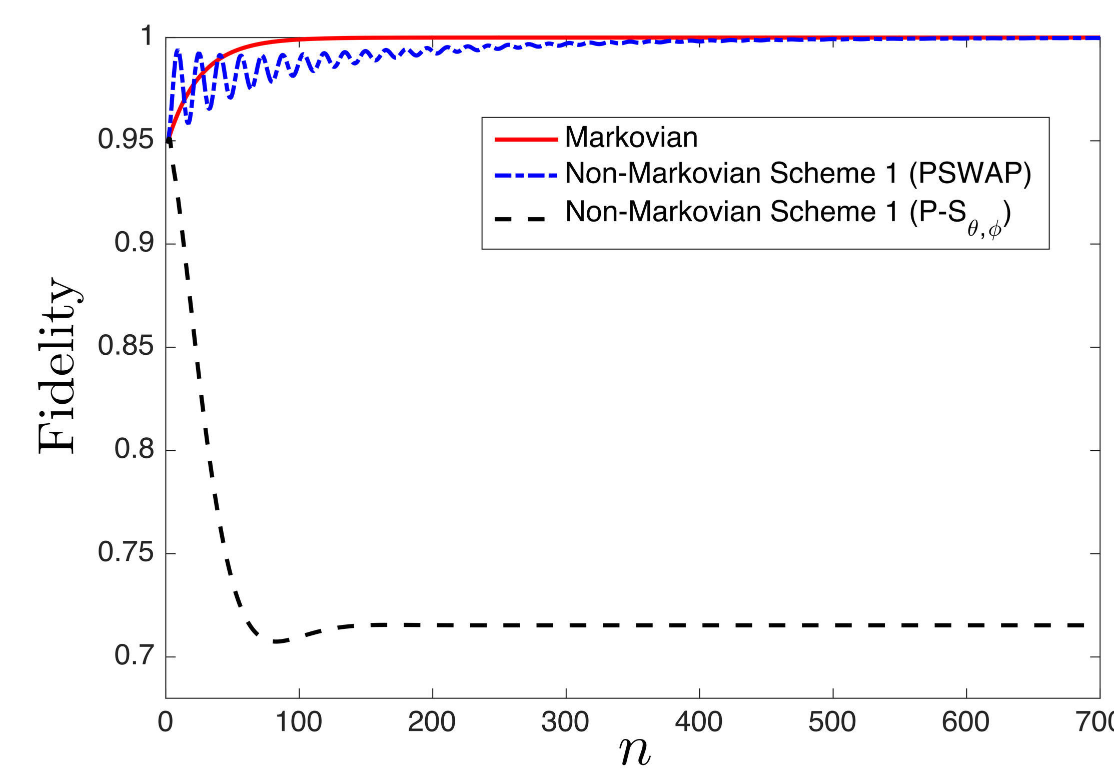
- Additional ancilla-ancilla interaction unitary:
$$U_{B_{n-1}B_n}(\delta) = (\cos\delta)\mathbb{1}_{4\times4} + i(\sin\delta)S_{4\times4}$$
- Ratio test provide conditional proof towards the goal.
- Suport for ratio test:
  (i) ***First Step:*** for a particular initial state of the ancilla homogenization happens for all initial system states.
  (ii) ***Second Step:*** generalize the first step to an arbitrary initial ancilla state by rotation and scaling operation.



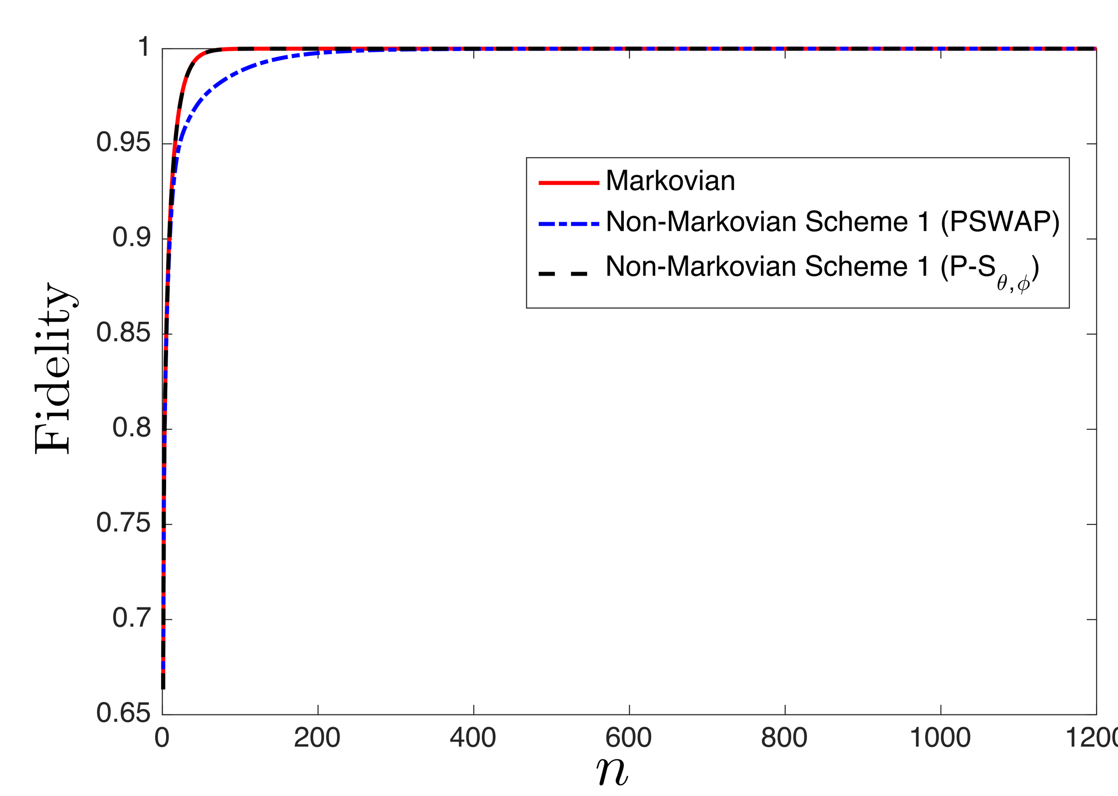


3. Non-Markovian Collisional Model for P-$S_{\theta,\phi}$

- Intra-ancilla interaction is Partial $S_{\theta,\phi}$ (P-$S_{\theta,\phi}$):
$$U_{B_{n-1}B_n}(\delta) = (\cos\delta)\mathbb{1}_{2\times2}\otimes\mathbb{1}_{2\times2} + i(\sin\delta)S_{\theta,\phi}.$$
$$S_{\theta,\phi} = \frac{1}{2}(\mathbb{1}_{2\times2}\otimes\mathbb{1}_{2\times2} + \sigma_z\otimes\sigma_z + \cos\theta(\sigma_z\otimes\mathbb{1} - \mathbb{1}\otimes\sigma_z) + \sin\theta\cos\phi(\sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y) + \sin\theta\sin\phi(\sigma_y\otimes\sigma_x - \sigma_x\otimes\sigma_y))$$



Thermalization



Conclusions

- PSWAP is unique for Homogenization in Markovian CM
- Homogenization happens universally in Non-Markovian CM – for PSWAP
- P-$S_{\theta,\phi}$ breaks universality and uniqueness
- P-$S_{\theta,\phi}$ speeds up Homogenization/thermalization

References

[1] M. Ziman, *et al.*, *Phys. Rev. A.* **65**, 042105 (2002)

[2] V. Scarani, *et al.*, *Phys. Rev. Lett.* **88**, 097905 (2002)