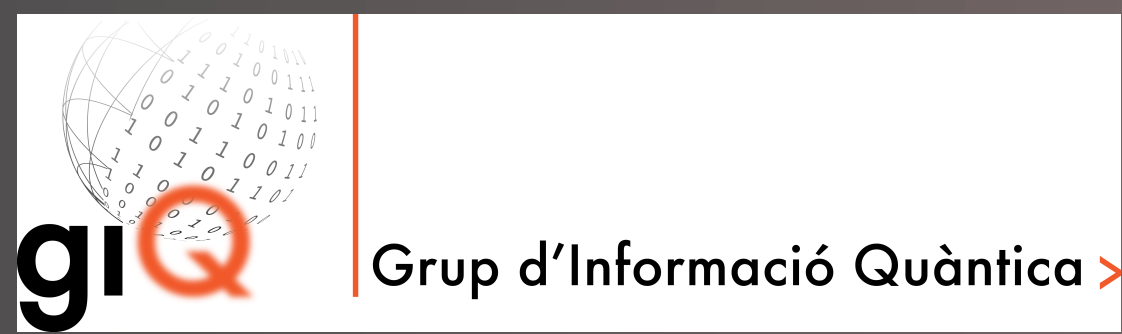


# Everything you wanted to know about the second law (but were afraid to ask)



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**Q1: How many second laws exist in thermodynamics?**

- (a) 1 (b) 4 (c)  $\infty$

**Q2: How do you define thermodynamic entropy microscopically?**

- (a)  $S = k_B \ln W(x)$  (b)  $S = -k_B \text{tr}\{\rho \ln \rho\}$  (c)  $S = k_B \sum_x p(x) [-\ln p(x) + \ln W(x)]$

**Q3: Why does entropy increase?**

- (a) because of incomplete information (“coarse-graining”)  
(b) because of a low entropy state in the past  
(c) because of weak forces that break time-reversal symmetry

**Q4: What is entropy production  $\Sigma$ ?**

- (a)  $\Delta S_S - \frac{Q}{T}$  (b)  $\Delta S_{\text{tot}}$  (c)  $D[p(\gamma) \| p_{\text{tr}}(\gamma^\dagger)]$

**Q5: Does a pure state of an isolated many-body system satisfy  $\Delta S_{\text{tot}} \geq 0$  when initialized in a low entropy state?**

- (a) Yes, always! (b) No, never! (c) Most of the times!

**Q6: For which states can you rigorously prove  $\Delta S_{\text{tot}} \geq 0$  for any (even driven) isolated system?**

- (a) For all states satisfying  $\sum_x p(x) [-\ln p(x) + \ln W(x)] = -\text{tr}\{\rho \ln \rho\}$  (b) For all Gibbs states! (c) For all pure states!

**Q7: What is Clausius’ inequality?**

- (a)  $\Delta S_S - \frac{Q}{T} \geq 0$  (b)  $\Delta S_S - \int \frac{dQ}{T} \geq 0$  (c)  $\Delta S_S - \frac{Q}{T} \leq 0$

**Q8: When is Clausius’ inequality identical to the second law?**

- (a) When the bath stays approximately thermal. (b) Always! (c) Never, it’s a completely different inequality.

**Q9: What does the condition of local detailed balance imply?**

- (a) No currents at equilibrium. (b) The steady state is a Gibbs state. (c) Non-negative entropy production rate.

**Q10: Order the following five terms in increasing magnitude for the initial system-bath state  $\rho_S(t_0) \otimes e^{-\beta_0 H_B} / Z_B$ ?**

- (a) 0 (b)  $\Delta S_{\text{tot}}$  (c)  $\Delta S_S + \Delta S_B$  (d)  $\Delta S_S - \int \frac{dQ}{T}$  (e)  $\Delta S_S - \frac{Q}{T_0}$

## Answers and Remarks

- Q1: (a)  
Q2: (c), see von Neumann & Wigner, recently revived by Šafránek, Deutsch & Aguirre  
Q3: (b)  
Q4: (b)  
Q5: (c), see von Neumann’s famous  $H$ -theorem  
Q6: (a)  
Q7: (b)  
Q8: (a)  
Q9: (c)  
Q10: (a)  $\leq$  (b)  $\leq$  (c)  $\leq$  (d)  $\leq$  (e)

## Further Reading

- Tutorial:** P. Strasberg & A. Winter, *First and Second Law of Quantum Thermodynamics: A Consistent Derivation Based on a Microscopic Definition of Entropy*, Phys. Rev. X Quantum **2**, 030202 (2021)  
**Resurrecting Clausius:** P. Strasberg, M. G. Díaz & A. Riera-Campenya, *Clausius inequality for finite baths reveals universal efficiency improvements*, Phys. Rev. E Lett. **104**, L022103 (2021).  
**Master equation approach:** A. Riera-Campenya, A. Sanpera & P. Strasberg, *Quantum systems correlated with a finite bath: Nonequilibrium dynamics and thermodynamics*, Phys. Rev. X Quantum **2**, 010340 (2021).  
**Book:** P. Strasberg, *Quantum Stochastic Thermodynamics: Foundations and Selected Applications*, Oxford University Press (2022)

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