Everything you wanted to know about the second law (but were afraid to ask)

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Q1: How many second laws exist in thermodynamics?

(a) 1 (b) 4 (c) ∞

Q2: How do you define thermodynamic entropy microscopically?

(a) $S = k_B \ln W(x)$ (b) $S = -k_B \mathrm{tr}\{\rho \ln \rho\}$ (c) $S = k_B \sum_x p(x)[-\ln p(x) + \ln W(x)]$

Q3: Why does entropy increase?

(a) because of incomplete information ("coarse-graining")
(b) because of a low entropy state in the past
(c) because of weak forces that break time-reversal symmetry

Q4: What is entropy production Σ?

(a) $\Delta S_S - \frac{Q}{T}$ (b) ΔS_{tot} (c) $D[p(\boldsymbol{\gamma})\|p_{\text{tr}}(\boldsymbol{\gamma}^\dagger)]$

Q5: Does a pure state of an isolated many-body system satisfy $\Delta S_{\text{tot}} \geq 0$ when initialized in a low entropy state?

(a) Yes, always! (b) No, never! (c) Most of the times!

Q6: For which states can you rigorously prove $\Delta S_{\text{tot}} \geq 0$ for any (even driven) isolated system?

(a) For all states satisfying $\sum_x p(x)[-\ln p(x) + \ln W(x)] = -\mathrm{tr}\{\rho \ln \rho\}$ (b) For all Gibbs states! (c) For all pure states!

Q7: What is Clausius' inequality?

(a) $\Delta S_S - \frac{Q}{T} \geq 0$ (b) $\Delta S_S - \int \frac{\text{đ}Q}{T} \geq 0$ (c) $\Delta S_S - \frac{Q}{T} \leq 0$

Q8: When is Clausius' inequality identical to the second law?

(a) When the bath stays approximately thermal. (b) Always! (c) Never, it's a completely different inequality.

Q9: What does the condition of local detailed balance imply?

(a) No currents at equilibrium. (b) The steady state is a Gibbs state. (c) Non-negative entropy production rate.

Q10: Order the following five terms in increasing magnitude for the initial system-bath state $\rho_S(t_0) \otimes e^{-\beta_0 H_B}/Z_B$?

(a) 0 (b) ΔS_{tot} (c) $\Delta S_S + \Delta S_B$ (d) $\Delta S_S - \int \frac{\text{đ}Q}{T}$ (e) $\Delta S_S - \frac{Q}{T_0}$

Answers and Remarks

Q1: (a)
Q2: (c), see von Neumann & Wigner, recently revived by Šafránek, Deutsch & Aguirre
Q3: (b)
Q4: (b)
Q5: (c), see von Neumann's famous H-theorem
Q6: (a)
Q7: (b)
Q8: (a)
Q9: (c)
Q10: (a) $\leq$ (b) $\leq$ (c) $\leq$ (d) $\leq$ (e)

Further Reading

Tutorial: P. Strasberg & A. Winter, *First and Second Law of Quantum Thermodynamics: A Consistent Derivation Based on a Microscopic Definition of Entropy*, Phys. Rev. X Quantum **2**, 030202 (2021)
Resurrecting Clausius: P. Strasberg, M. G. Díaz & A. Riera-Campeny, *Clausius inequality for finite baths reveals universal efficiency improvements*, Phys. Rev. E Lett. **104**, L022103 (2021).
Master equation approach: A. Riera-Campeny, A. Sanpera & P. Strasberg, *Quantum systems correlated with a finite bath: Nonequilibrium dynamics and thermodynamics*, Phys. Rev. X Quantum **2**, 010340 (2021).
Book: P. Strasberg, *Quantum Stochastic Thermodynamics: Foundations and Selected Applications*, Oxford University Press (2022)

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