Dimensional enhancements in a quantum battery with imperfections

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Abstract

We show that the average power output of a quantum battery based on a quantum interacting spin model, charged via a local magnetic field, can be enhanced with the increase of spin quantum number, thereby exhibiting dimensional advantage in quantum batteries. Moreover, we report that such dimensional advantages persist even when the battery Hamiltonian has some defects or when the initial battery state is prepared at finite temperature.

Quantum battery

A quantum battery is modeled as a finite number of quantum-mechanical interacting systems in d dimensions, governed by a Hamiltonian, H^j_B, with j being the spin quantum number, indicating the dimension of the battery. To charge the system, a local magnetic field, governed by the Hamiltonian, H^j_c, is applied to each subsystem.

Models and figure of merits

$$H_B^j = \frac{1}{2} h \sum_{k=1}^{N} S_k^z + \frac{1}{4} \sum_{k=1}^{N-1} J_k [(1+\gamma) S_k^x \otimes S_{k+1}^x + (1-\gamma) S_k^y \otimes S_{k+1}^y],$$

$$H_B^j(\phi) = \sum_{k=1}^{N-1} J_k [\cos\phi (\vec{S}_k \cdot \vec{S}_{k+1}) + \sin\phi (\vec{S}_k \cdot \vec{S}_{k+1})^2] + \frac{h}{2} \sum_{k=1}^{N} S_k^z.$$

$$H_c^j = \frac{\omega}{2} \sum_{k=1}^{N} S_k^x, \qquad H_c^j = \omega \sum_{k=1}^{N} \frac{S_k^x}{2} + \frac{(S_k^x)^2}{4},$$

$$W(t) = \mathrm{Tr}[H_0 \rho(t)] - \mathrm{Tr}[H_0 \rho(t=0)],$$

$$P_{\max} = \max_t \frac{W(t)}{t}, \qquad \mathcal{E}(t) = E_B(t) - \min_{U_B} \mathrm{Tr}[H_B U_B \rho(t) U_B^\dagger].$$

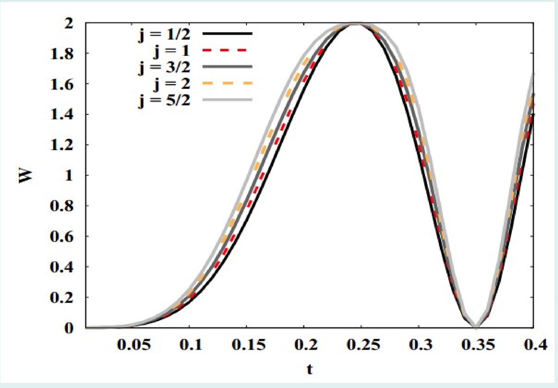
Analytical results for N = 2

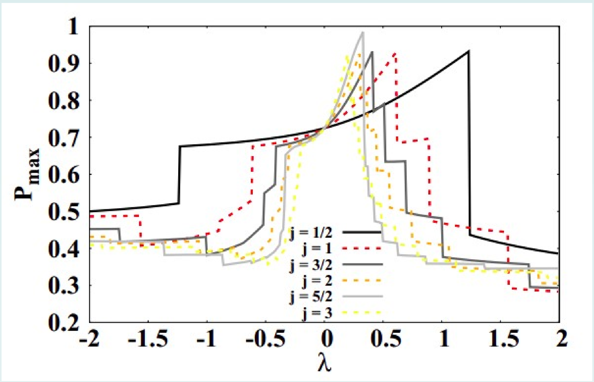
If the initial state of the battery is the ground state, i.e., the zero-temperature state of the spin-1 transverse XX model, the maximum average power of the battery, Pmax is higher than that of the XX model consisting of spin-1/2 particles provided the interactionstrength between the sites is weak.

$$W_{\text{final}} = (a \cos t + b\lambda \sin^2 t),$$

$$P^{j=1}(t) = a' - b' \cos t + \cos t(-b' - c' \cos t) + [-2(1-a') - c' \cos t + (1-a') \cos 2t] \cos 2t + 0.05 \sin^2 t - \frac{\sin t \sin^2 t}{4} + (1-a') \sin^2 t, \quad (7)$$

Enhanced battery power with the increase of spin





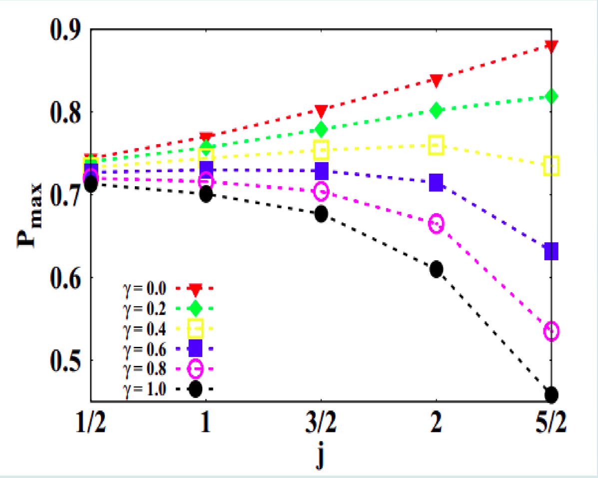
Although the hierarchy among power is proven by comparing j = 1/2 and 1, it can be shown to be true for other higher dimensional systems as well, for small values of λ = J/|h|. the work output is oscillating with time, although its maximum value is fixed. Since W (t) is a strictly increasing function of time between the initial time and the time when it reaches its maximum value which leads to the power generation.

Small value of J allows the system to point towards the z-direction. On the other hand, charging field is in the x-direction. Aligning the spins along the direction of the charging field requires more energy for driving the system out of equilibrium than the scenario when J is considerably large.

Maximum values of interaction strength, for which P_{max} reaches its maximum for fixed dimensions of spins, j.

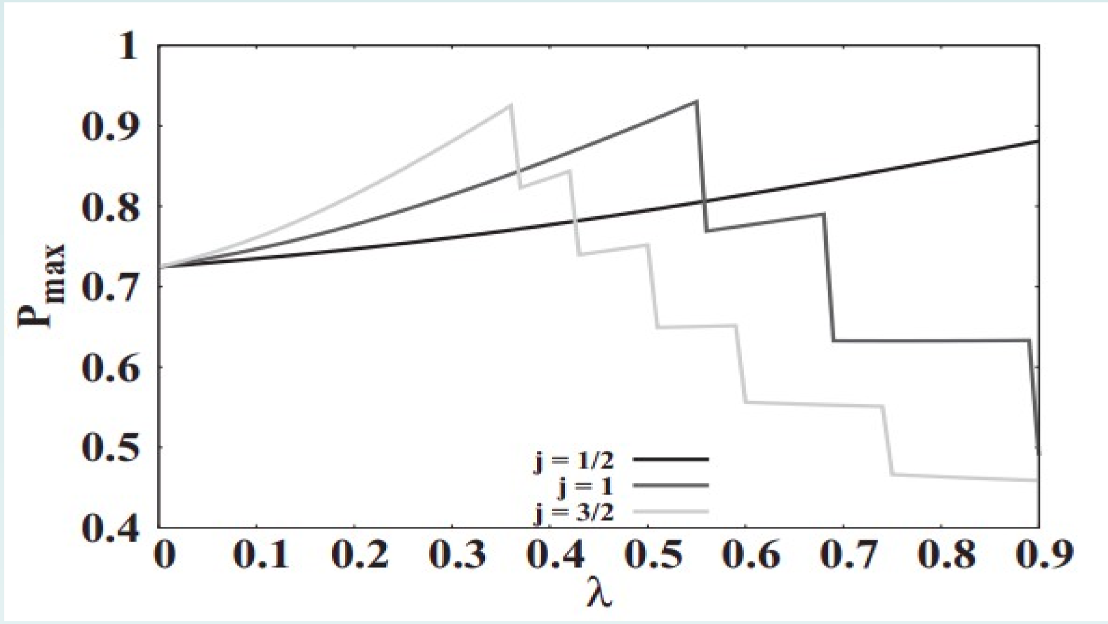
j	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$\lambda_{\max}(\gamma = 0)$	1.23	0.61	0.41	0.30	0.33	0.20
$\lambda_{\max}(\gamma = 0.2)$	1.15	0.51	0.33	0.25	0.26	0.17

Anisotropy dependence



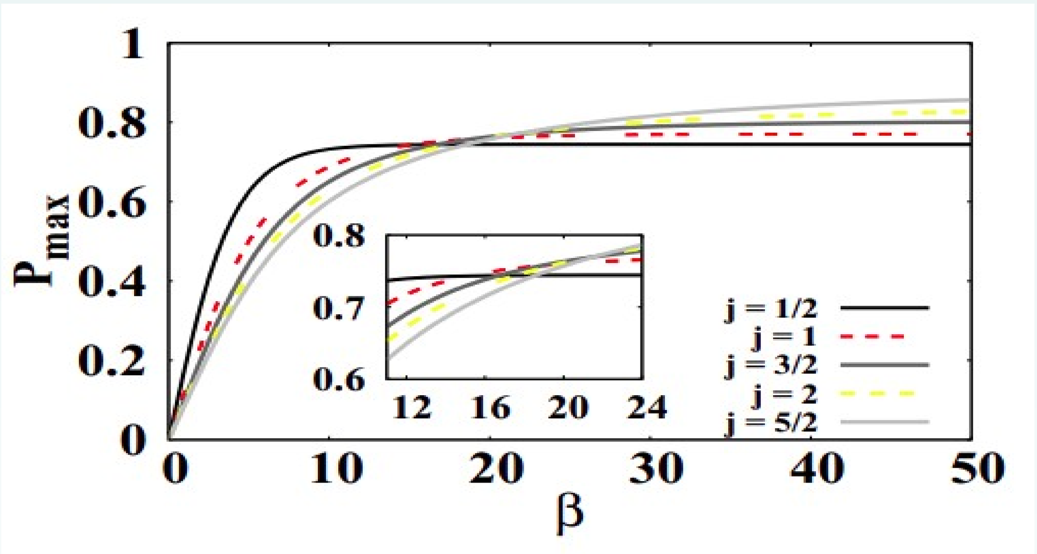
Up to the numerical accuracy, we report that $\gamma^{0.01}_{critical}$ =0.98 while $\gamma^{0.1}_{critical}$=0.8 for all values of j =3/2. Moreover, we fix the interaction strength, say, λ = 0.2, and see the behavior of Pmax with j for a different anisotropy.

Higher system size



The performance remains qualitatively similar even if we consider the battery Hamiltonian with a large number of spins which we check for small dimensions with N = 6.

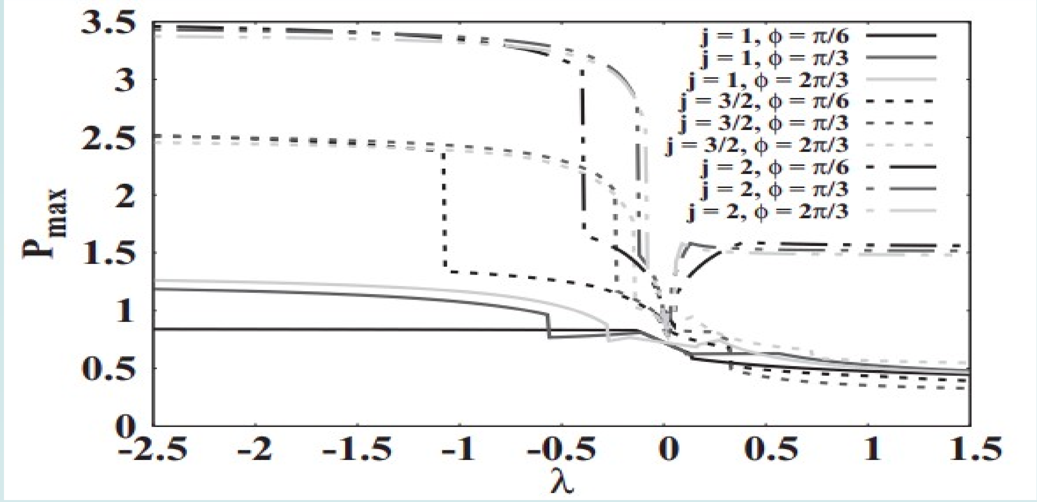
Temperature-induced power generation



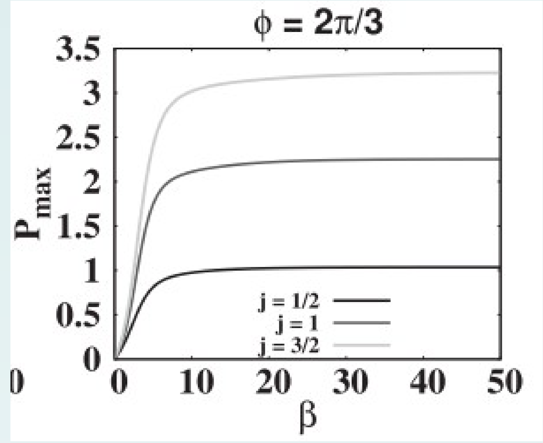
For small value of T, we notice that higher dimensional spins give a larger amount of power output than that of the low-dimensional systems provided the interaction strength is weak and positive in the XX model.

j	1	$\frac{3}{2}$	2	$\frac{5}{2}$
β_{critical}	15.5	18.5	20	21

BBH model

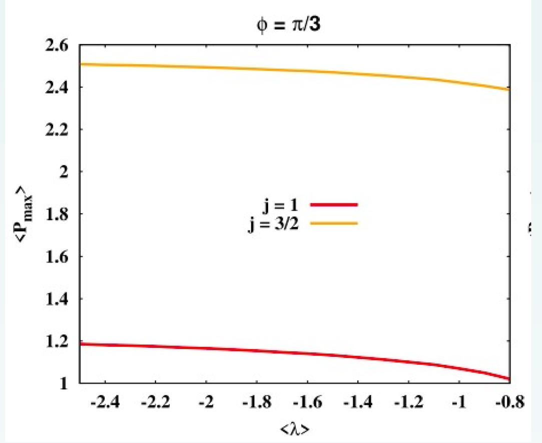


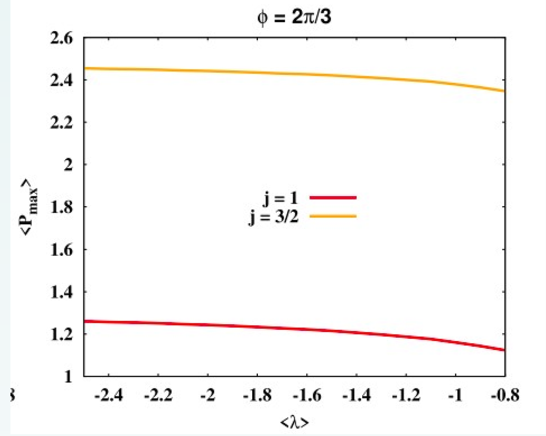
We observe two types of improvements in the performance of the battery, one is due to the increase of spin quantum number and other one is for the increase of the interaction strength in the negative direction. It can be argued that for a fixed φ, Pmax saturates to a finite value even in arbitrary large dimension, thereby exhibiting quantum gain in the battery over its classical counterparts.



The behavior of P_{max} again turns out to be phase dependent and the trade-off between phases and dimension still exists.

Disoredered system





We find that with moderately high standard deviation, quenched averaged power $<P_{max}>$of the spin- 3/2 BBH model is significantly higher than that of the spin-1 case. Here standard deviation = 0.2.

References

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