Daemonic ergotropy and non-classical thermalization via the quantum SWITCH

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Setup

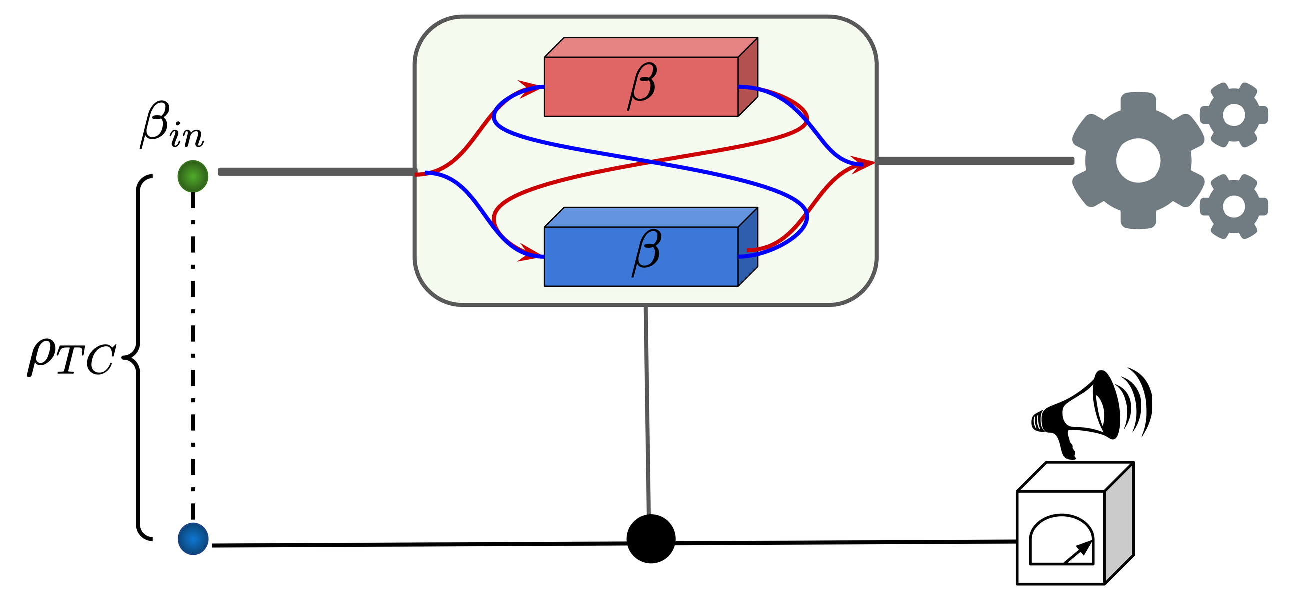

Figure 1: The work medium and control system are initially in a joint state ρ_{TC}, and the work medium is locally at temperature β_{in}, i.e., $c[\rho_{TC}] = \tau_{\beta_{in}}$. The work medium undergoes action of N thermalizing maps T_β (at the figure $N = 2$) at temperature β whose application order is controlled by the quantum SWITCH. Thereafter, the control system is measured, and the ergotropic work is extracted from the resulting state of the work medium.

General scenario: Working medium at initial temperature β_{in}. No ergotropic work can be extracted from it.

Output state after thermalization with reservoirs at temperature β placed in a well-defined or chosen by tossing a coin causal order:

$$\sigma = (T_\beta \circ \ldots \circ T_\beta)[\tau_{\beta in}] = \tau_\beta. \quad (1)$$

No ergotropic work can be extracted from it as well.

Quantum SWITCH consists of a N-level system (usually $N = 2$) coherently controlling the occurrence order of quantum operations, hence, making it causally indefinite.

It takes the joint state ρ_{TC} and applies the maps T_β to the work medium in an order determined by the state of the control system,

$$\sigma_{TC} = \mathcal{S}(T_\beta, \cdots, T_\beta)[\rho_{TC}] = \sum_{i_1,\cdots,i_N} S_{i_1\cdots i_N}(\rho_{TC}) S^\dagger_{i_1\cdots i_N}, \quad (2)$$

where the Kraus operators

$$S_{i_1\ldots i_N} = \underbrace{(U_{i_N}\ldots U_{i_2}U_{i_1}) \otimes |0\rangle\langle 0|}_{\text{order } 1\to\ldots\to N-1\to N} + \ldots + \underbrace{(U_{i_{N-1}}\ldots U_{i_1}U_{i_N}) \otimes |N-1\rangle\langle N-1|}_{\text{order } N\to 1\to\ldots\to N-1} \quad (3)$$

consist of terms, each realizing one of the cyclic orders of the maps T_β with a set of Kraus operators $\{U_i\}$. Seen as an ancilla the control system allows to prepare a state

$$\sigma_a = p_a^{-1} \mathrm{Tr}_C[(I \otimes \Pi_a)\sigma_{TC}], \quad (4)$$

of the work medium by performing a projective measurement $\{\Pi_a\}$ on the control system, where a corresponds to its outcome. The corresponding extractable work is given by the daemonic ergotropy

$$W_d(\sigma_{TC}) = \sum_a p_a \mathcal{W}(\sigma_a) \geq 0, \quad (5)$$

which is an average ergotropy that can be extracted from post-selected states of the work medium after the control system is measured. Therefore, **if the reservoirs are coupled in an indefinite causal order, the output state of the work medium is not necessarily passive**.

Uncorrelated work medium and control

We focus on a two-level work medium evolving under Hamiltonian $\hat{H}_S = \mathrm{Diag}[0, 1]$. In the usual quantum-SWITCH-scenarios, the work medium and control system are uncorrelated,

$$\hat{\rho}_{in} = \tau_{\beta_{in}} \otimes |\gamma_+\rangle\langle\gamma_+|, \quad (6)$$

where $|\gamma_+\rangle = \frac{1}{\sqrt{N}}\sum_i |i\rangle$. After the work medium is thermalized with the reservoirs via the quantum SWITCH, one can perform a yes/no measurement w.r.t. $|\gamma_+\rangle$ and activate the state of the work medium,

$$W_d(\sigma_{TC}) = \left(1 - \frac{1}{N}\right)\frac{\max\{0, e^{-2\beta} - e^{-\beta_{in}}\}}{Z_\beta^2 Z_{\beta_{in}}}, \quad (7)$$

where Z_β is the the partition function. Therefore, increasing number N of controlled T_β *increases extractable work*. However, it is non-zero only in a certain region of temperatures (β_{in}, β) established by the **temperature bound**,

$$\beta_{in} > 2\beta. \quad (8)$$

Genuinely classical correlations

We ask whether the temperature bound (8) can be beaten. Instead of increasing the number N of reservoirs, we fix $N = 2$ and assume that work medium and control are initially classically correlated,

$$\rho_{TC} = \frac{1}{Z_{\beta_{in}}}\Big(|0\rangle\langle 0| \otimes |+\rangle\langle +| + e^{-\beta_{in}}|1\rangle\langle 1| \otimes |-\rangle\langle -|\Big), \quad (9)$$

so that locally the work medium remains in a thermal state $\tau_{\beta_{in}}$. In this case, extractable work from the output can increase,

$$W_d(\sigma) = \frac{1}{2Z_\beta^2 Z_{\beta_{in}}}\max\{0, e^{-2\beta} - e^{-\beta^{in}} + 2e^{-(2\beta+\beta_{in})}\}, \quad (10)$$

and the temperature bound is typically shifted and vanishes iff $\beta < \frac{1}{2}\ln 3$,

$$\ln(e^{\beta_{in}} + 2) > 2\beta. \quad (11)$$

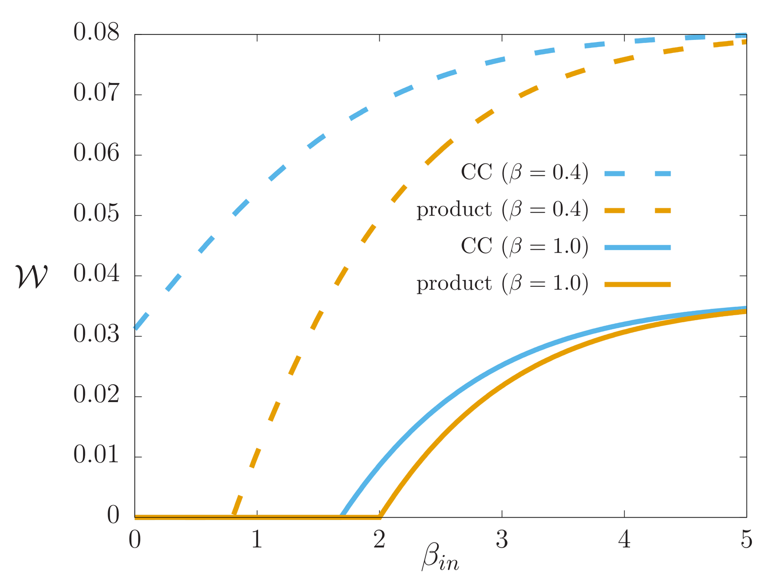

Figure 2: Dependence of ergotropy on the initial temperature.

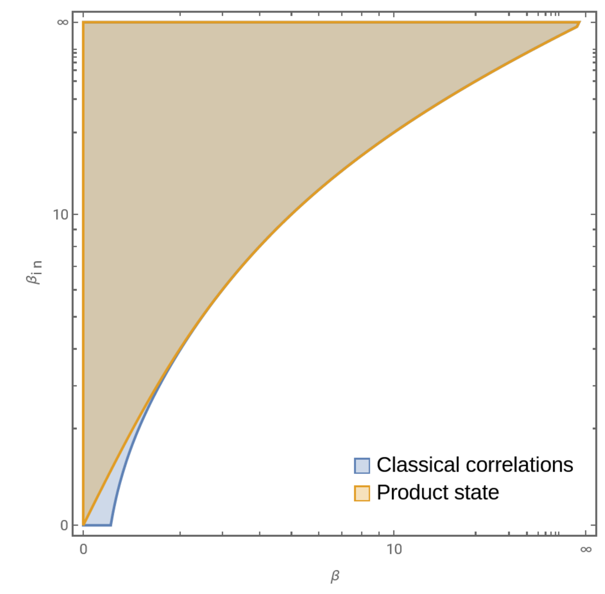

Figure 3: Regions of the output non-zero ergotropy.

Quantum correlations

Now we allow the work medium and control to be initially entangled,

$$|T_\alpha(\beta_{in})\rangle = \frac{1}{\sqrt{Z_{\beta_{in}}}}\Big(|0\psi\rangle + e^{-\frac{\beta_{in}}{2}}|1\psi^\perp\rangle\Big), \quad (12)$$

where α parameterizes the pure qubit state $|\psi\rangle$, so that $\rho_{TC} = |T_\alpha(\beta_{in})\rangle\langle T_\alpha(\beta_{in})|$, and locally the work medium remains in a thermal state $\tau_{\beta_{in}}$. We ask which purification α leads to the maximal ergotropy $\mathcal{W}(\sigma_{TC})$ of the output state and find that it depends on the temperature bound (8),

$$\mathcal{W}(\sigma_{TC}) = \begin{cases} \frac{1}{2}\tanh\left(\frac{\beta}{2}\right)\left(\sqrt{1 + \frac{1}{4}\sinh^{-2}(\beta)\cosh^{-2}\left(\frac{\beta_{in}}{2}\right)} - 1\right), & \beta_{in} \leq 2\beta \\ \frac{1}{2Z_\beta^2 Z_{\beta_{in}}}\max\{0, e^{-2\beta} - e^{-\beta^{in}} + 2e^{-(2\beta+\beta_{in})}\}, & \beta_{in} > 2\beta \end{cases} \quad (13)$$

For $\beta_{in} \leq 2\beta$, the optimal $\alpha = 0, 1$, so that $\psi = |0/1\rangle$, and the maximal work is purely coherent. Importantly, it does not vanish anywhere in (β, β_{in}), so the temperature bound (8) is **completely beaten**. For $\beta_{in} > 2\beta$, the optimal $\alpha = \frac{1}{2}$, so that $\psi = |+\rangle$, and the maximal work is purely incoherent and coincides with the ergotropy (10) for the classical correlations.

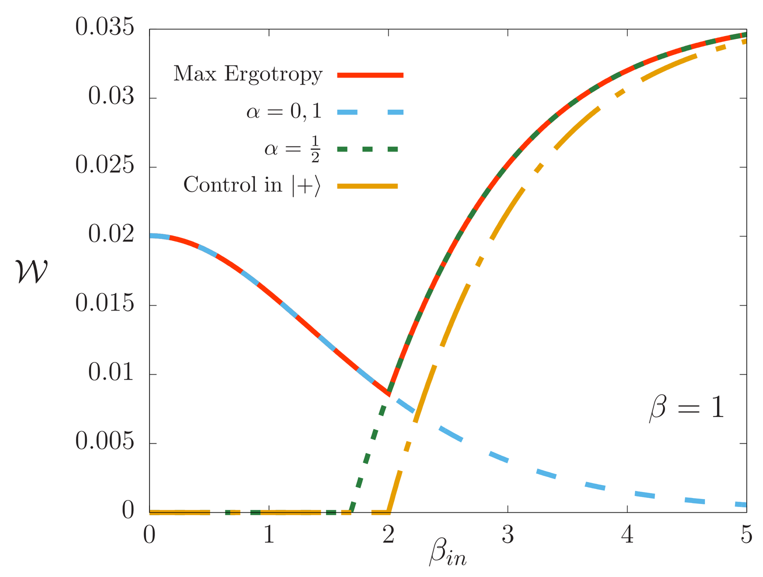

Figure 4: Dependence of ergotropy on the initial temperature.

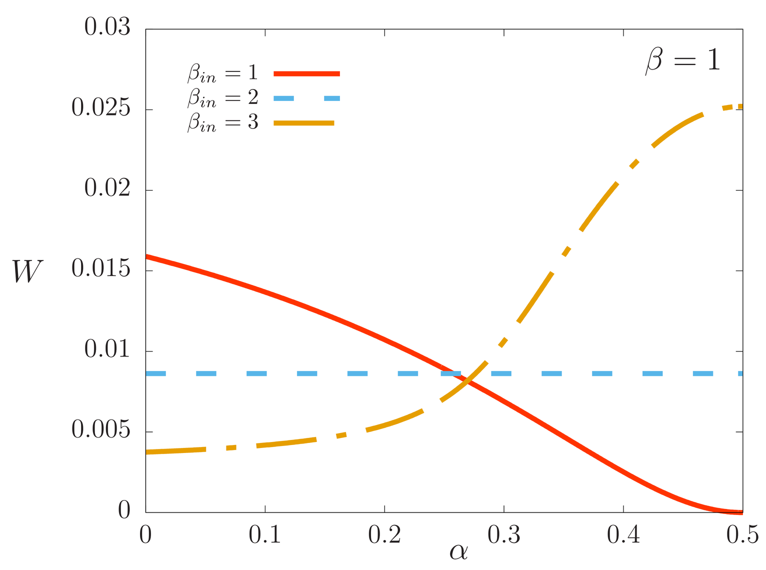

Figure 5: Dependence of ergotropy on the purification.

The same results can be derived also for the work medium and control initially sharing quantum discord associated with measurements of former.

Conclusions

Thermalization with reservoirs put into an indefinite causal order via the quantum SWITCH allows to activate even an initially thermal state of the system. However, the ergotropic gain depends on the temperatures of the system and maps and can be zero for a high initial temperature of the latter. We have shown that quantum correlations between the system and control allow to remove this bound and extract a non-zero work by any temperatures of the system and maps.