Quantum Energetics: Connecting an Open- and a Closed-system approach

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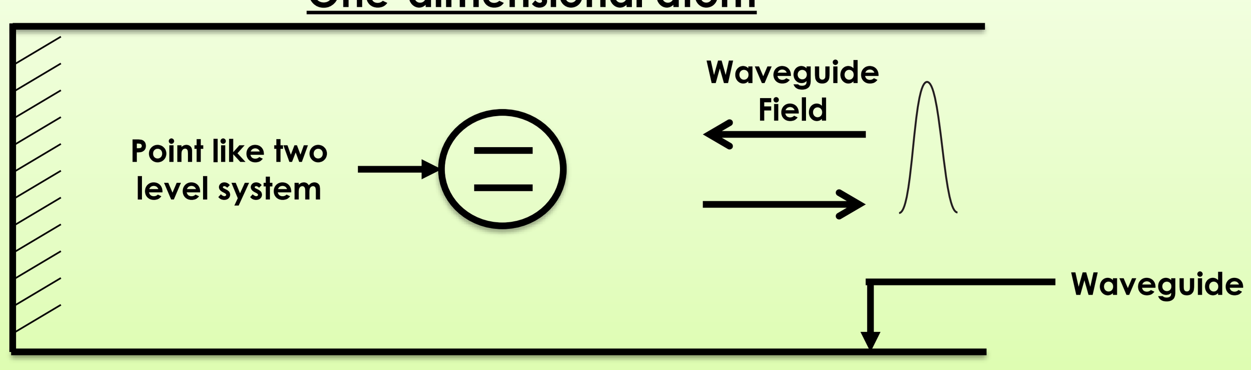
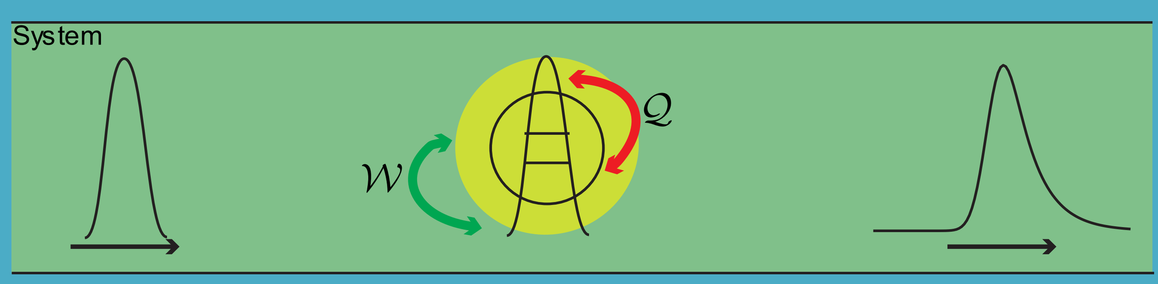
Motivations and Introduction

- The amount of information available about systems interacting changes the energetic description that is used.
- The standard open-system energetics is compared to closed-system energetics.
- We consider a paradigmatic and experimentally testable system of quantum optics that can be described by both the open- and closed-system perspectives, the one-dimensional (1D) atom: a qubit interacting with an electromagnetic field propagating in a 1D waveguide.

One-dimensional atom



1D Atom in Waveguide



$$H_S = \hbar\omega_S\sigma_+\sigma_- \qquad H_f = \sum_k \hbar\omega_k a_k^\dagger a_k \qquad V_{Sf} = i\hbar g \sum_k \left(\sigma_+ a_k - \sigma_- a_k^\dagger\right)$$

- The state is separated into a local part and correlations at each time:

$$\rho_{Sf}(t) = \rho_S(t) \otimes \rho_f(t) + \chi(t)$$

Correlation Matrix

- Each sub-system exerts a unitary drive on the other, i.e., effective drive: $\mathcal{H}_k(t) = \mathrm{Tr}_{l\neq k}\{V_{Sf}\rho_l(t)\}$

$$\dot{\rho}_k = -\frac{i}{\hbar}\left[H_k + \mathcal{H}_k(t), \rho_k\right] - \frac{i}{\hbar}\mathrm{Tr}_{k\neq l}\{[V_{Sf}, \chi(t)]\}$$

Unitary Dynamics — Completely non-unitary dynamics

Closed System Energetics

- Internal Energy Flux: $\dot{\mathcal{U}}_k = \mathrm{Tr}_k\{H_k\dot{\rho}_k\} = \dot{\mathcal{U}}_k^{\otimes} + \dot{\mathcal{U}}_k^{\chi}$

$$\dot{\mathcal{W}}_k(t) = -\frac{i}{\hbar}\mathrm{Tr}_k\{[H_k, \mathcal{H}_k(t)]\rho_k\} \qquad \dot{\mathcal{Q}}_k(t) = -\frac{i}{\hbar}\mathrm{Tr}_k\{H_k \mathrm{Tr}_{l\neq k}\{[V_{Sf}, \chi(t)]\}\}$$

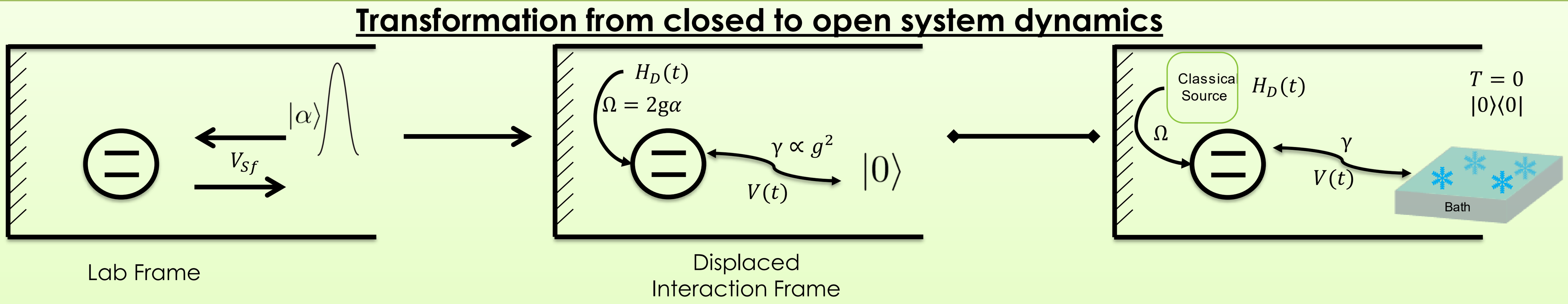
(Unitary dynamics) (Non-unitary dynamics)

- Coupling Energy Flux: $\dot{\mathcal{V}}_{Sf} = \mathrm{Tr}\{V_{SF}\dot{\rho}_{Sf}\} = \dot{\mathcal{V}}_{Sf}^{\otimes} + \dot{\mathcal{V}}_{Sf}^{\chi}$

BALANCE EQUATIONS

0	=	$\dot{\mathcal{U}}_S$	+	$\dot{\mathcal{U}}_f$	+	$\dot{\mathcal{V}}_{Sf}$
		=		=		=
0	=	$\dot{\mathcal{W}}_S$: $-\frac{i}{\hbar}\mathrm{Tr}\{[H_S, \mathcal{H}_S(t)]\rho_S(t)\}$	+	$\dot{\mathcal{W}}_f$: $-\frac{i}{\hbar}\mathrm{Tr}\{[H_f, \mathcal{H}_f(t)]\rho_f(t)\}$	+	$\dot{\mathcal{V}}_{Sf}^{\otimes}$: $-\frac{i}{\hbar}\mathrm{Tr}\{[V_{Sf}, H_S + H_f](\rho_S(t)\otimes\rho_f(t))\}$
		+		+		+
0	=	$\dot{\mathcal{Q}}_S$: $-\frac{i}{\hbar}\mathrm{Tr}\{[H_S, V_{Sf}]\chi(t)\}$	+	$\dot{\mathcal{Q}}_f$: $-\frac{i}{\hbar}\mathrm{Tr}\{[H_f, V_{Sf}]\chi(t)\}$	+	$\dot{\mathcal{V}}_{Sf}^{\chi}$: $-\frac{i}{\hbar}\mathrm{Tr}\{[V_{Sf}, H_S + H_f]\chi(t)\}$

Transformation from closed to open system dynamics

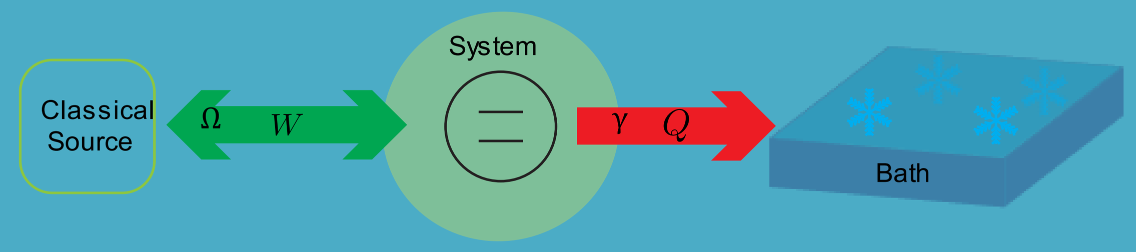


$$\dot{\rho_S}(t) = -\frac{i}{\hbar}\left[H_S + H_D(t), \rho_S(t)\right] - \frac{i}{\hbar}\mathrm{Tr}_f\{[V(t), \rho_{Sf}(t)]\}$$

Optical Bloch Equation Dissipator

$$-\frac{i}{\hbar}\mathrm{Tr}_f\{[V(t), \rho_{Sf}(t)]\} \approx \gamma\mathcal{D}[\rho_S(t)] \equiv \gamma\left(\sigma_-\rho_S(t)\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_S(t)\}\right)$$

Open Quantum System Dynamics



- System and thermal bath → quantum, external source → classical
- The dynamics of the qubit is described by the optical Bloch equation (OBE) at 0 temperature where $(\omega_s, \omega_L \gg \Omega, \gamma)$

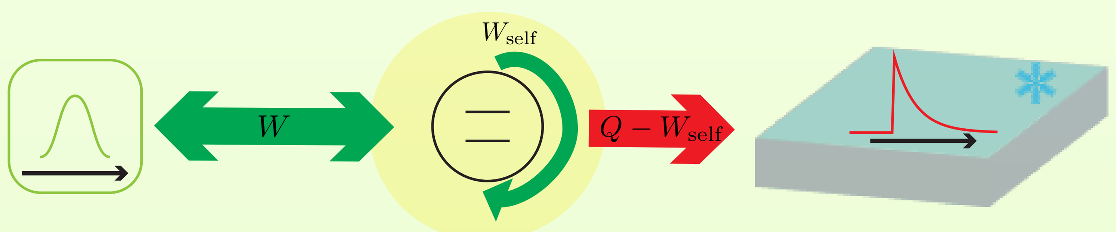
$$\dot{\rho_S}(t) = -\frac{i}{\hbar}\left[H_S + H_D(t), \rho_S(t)\right] + \gamma\mathcal{D}[\rho_S(t)]$$

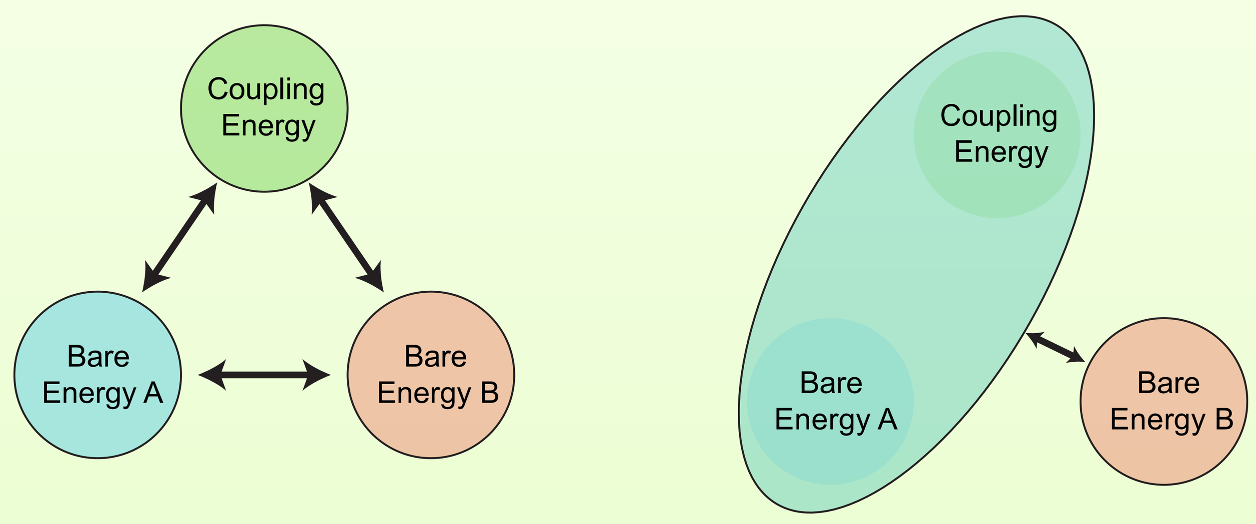
- The bath is given a special position as it models inaccessible degrees of freedom

Open/Standard System Energetics

- Internal energy of the qubit: $U(t) \equiv \mathrm{Tr}\{\rho_S(t)(H_S + H_D(t))\}$
- Work flux ≡ Energy input from external drive: $\dot{W}(t) \equiv \mathrm{Tr}\left\{\rho_S(t)\frac{d}{dt}H_D(t)\right\}$
- Heat flux ≡ Energy flux out of the thermal bath: $\dot{Q}(t) \equiv \gamma\mathrm{Tr}\{H_S\mathcal{D}[\rho_S(t)]\} + \gamma\mathrm{Tr}\{H_D(t)\mathcal{D}[\rho_S(t)]\}$

Comparison





- Dissipator of OBE includes a unitary component: $\mathcal{D}[\rho_S(t)] = \mathcal{D}_{\otimes}[\rho_S(t)] + \mathcal{D}_{\chi}[\rho_S(t)]$

$$\gamma\mathcal{D}_{\otimes}[\rho_S(t)] \equiv -\frac{i}{\hbar}\mathrm{Tr}_f\{[V(t), \rho_S(t)\otimes\rho_f(t)]\} = -\frac{i}{\hbar}[\mathcal{H}_S^s(t), \rho_S(t)]$$

- Self-Drive Hamiltonian is the drive term due to coupling with vacuum field: $\mathcal{H}_S^s(t) \equiv \mathrm{Tr}_f\{V(t)\rho_f(t)\}$
- Re-writing of the master equation: $\dot{\rho}_S(t) = -\frac{i}{\hbar}\left[H_S + H_D(t) + \mathcal{H}_S^s(t), \rho_S(t)\right] + \gamma\mathcal{D}_{\chi}[\rho_S(t)]$
- Work term due to the extra Hermitian part of the master equation ≡ Self-Work: $\dot{W}_{\mathrm{self}}(t) \equiv \mathrm{Tr}\left\{\rho_S(t)\frac{d}{dt}\mathcal{H}_S^s\right\}$

Work and Heat Relations

$$\dot{U}(t) = -\dot{\mathcal{U}}_f$$

$$\dot{W}(t) + \dot{W}_{\mathrm{self}}(t) = \dot{\mathcal{W}}_S(t) + \dot{\mathcal{V}}_{Sf}^{\otimes}(t) = -\dot{\mathcal{W}}_f(t) \qquad \dot{Q}(t) - \dot{W}_{\mathrm{self}}(t) = -\dot{\mathcal{Q}}_f(t)$$

Example: Spontaneous Emission Work

- Waveguide starts in vacuum: $H_D(t) = 0$
- Qubit initial state: $|\psi\rangle = \alpha|e\rangle + \beta|g\rangle$
- Here $\dot{W}(t) = 0$ but,

$$\dot{\mathcal{W}}_S(t) = -\dot{\mathcal{W}}_f(t) = \dot{W}_{\mathrm{self}}(t) = -\gamma\hbar\omega_S\left|\langle\sigma_-(t)\rangle\right|^2$$

Summary and Discussion

- **The standard internal energy is all the energy given out of the field but not all the energy taken in by the qubit due to the presence of the coupling energy. In the resonant case, the coupling energy is 0, so the standard description holds.**
- **Work provided by the field in the closed perspective, can be split into two terms: one coming from the drive $H_D(t)$, which is the only work captured by the open-system perspective, and one coming from the qubit self-drive $\mathcal{H}_S^s(t)$, which is taken as heat in the open-system perspective instead.**

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