Unified collision model of coherent and measurement-based quantum feedback

Based on collision models, we provide a simple treatment of coherent and measurement-based quantum feedback on an equal footing. arXiv:2204.00479

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1 Motivations

- Two main strategies for quantum feedback control: 'coherent feedback' (CF) and 'measurement-based feedback' (MF).
- In CF the controller is a quantum system which processes quantum information, in MF the controller processes classical information resulting from measurement outcomes;
- Comparison between CF and MF not fully developed. There are claims that CF is always superior [1] but feedback loop not always explicitly modelled in CF and rely on initialize the controller in an arbitrary quantum state [2].
- We use the framework of collision models to compare CF and MF under the same dynamical conditions.

2 Model

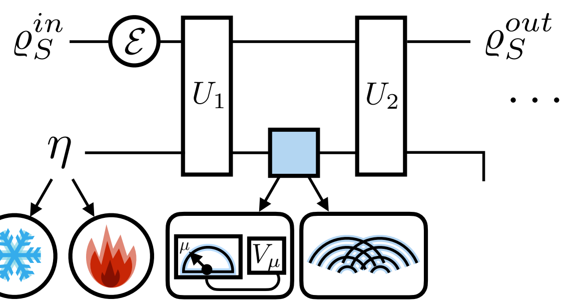

Figure: Schematic diagram for a single "feedback collision"; the dynamics is obtained upon repeated applications.

1. The system we want to control cannot be manipulated directly and is subject to noise, described by the quantum map $\mathcal{E}$.
2. The system and controller interact through a partial swap $U_s = \sqrt{\tau}\mathbb{I} - i\sqrt{1-\tau}\hat{S}$, where $\hat{S}$ is the swap gate and $0 \leq \tau \leq 1$.
3. **For CF:** a unitary operation, chosen from a set $\{V_j\}$, is performed on the controller;
   For MF: A non-destructive measurement is performed on the controller, with outcome μ, followed by a unitary V_μ, from the set $\{V_j\}$.
4. The system and controller interact again through U_s.
5. The state of the controller is refreshed to its initial state η.

3 Feedback cooling

Noisy (hot) controller: the controller is initialized in $\eta = \frac{\mathbb{I}}{d}$, with d Hilbert space dimension and we take $\mathcal{E}(\rho) = \lambda\rho + (1-\lambda)\frac{\mathbb{I}}{d}$.

- For CF, regardless of the in-loop single qudit unitary used, CF is incapable of counteracting the noise on the system and leads to the steady state $\rho_{CF}^{\text{ss}} = \frac{\mathbb{I}}{d}$.
- For MF, we consider an in-loop projective measurement in an arbitrary basis $\{|j\rangle, j = 0, \ldots, d-1\}$, after which a unitary applied to the controller maps all post-measurement states to the same state $|0\rangle$. Considering the averaged dynamics of this MF protocol, the system reaches the steady state

$$\rho_{MF}^{\text{ss}} = \frac{1}{d}\frac{d(1-\tau)+\tau-\lambda\tau^2}{1-\lambda\tau^2}|0\rangle\langle 0| + \sum_{j=1}^{d-1}\frac{1}{d}\frac{\tau-\lambda\tau^2}{1-\lambda\tau^2}|j\rangle\langle j|\,,$$

which for $\tau \neq 1$ has lower entropy than ρ_{CF}^{ss}.

Clean (cold) controller: the controller is initialized in $\eta = |0\rangle\langle 0|$.

- Two regimes: The steady-state (linear) entropy for MF is $S_{MF} = \frac{1}{2} - \frac{(\tau^2-1)^2}{2(\tau^2\lambda-1)^2}$, while for CF protocol we have $S_{CF} = \frac{1}{2} - \frac{8\tau^2(\tau-1)^2}{((1-2\tau)^2\lambda-1)^2}$. Comparing the two expressions, we find that $S_{MF} < S_{CF}$ for $\tau < 1/3$ and $S_{MF} > S_{CF}$ for $\tau > 1/3$.

4 Suppressing decay

The system is a qubit initialized in the excited state $|1\rangle$ and subject to decay, modelled by $\mathcal{E}$ with Kraus operators $E_0 = \sqrt{\gamma}|0\rangle\langle 1|$, $E_1 = \sqrt{1-\gamma}|1\rangle\langle 1| + |0\rangle\langle 0|$. The controller is initialized in $\eta = \frac{\mathbb{I}}{2}$.

- For CF, we restrict the in-loop unitaries to rotations $U = \cos\chi\mathbb{I} + i\sin\chi\sigma_y$. For $\tau < \frac{1}{2}$, the optimal protocol is obtained for $\chi = \frac{\pi}{2}$, yielding a stationary excited state probability

$$p_{CF}^{\text{ss}} = \frac{1-\tau}{2(\gamma-1)\tau-\gamma+2}\,.$$

- For MF, we measure the controller in the $\{|1\rangle, |0\rangle\}$ basis, do nothing if the result is $|1\rangle$ and apply an in-loop rotation with $\chi = \frac{\pi}{2}$ if the result is $|0\rangle$. The excited state probability is

$$p_{MF}^{\text{ss}} = \frac{2-\tau^2-\tau}{2(\gamma-1)\tau^2+2}\,.$$

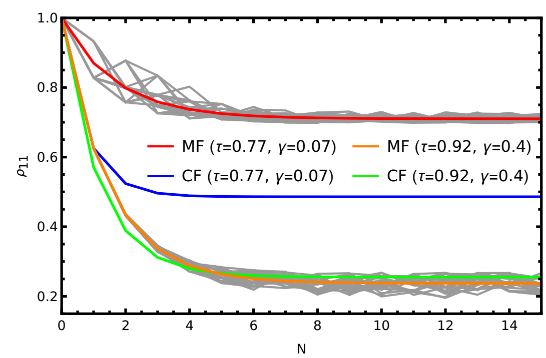

Figure: excited state probability against number of iterations.

References

[1] N. Yamamoto, "Coherent versus measurement feedback: Linear systems theory for quantum information," *Physical Review X*, vol. 4, no. 4, p. 041 029, 2014.

[2] K. Jacobs, X. Wang, and H. M. Wiseman, "Coherent feedback that beats all measurement-based feedback protocols," *New Journal of Physics*, vol. 16, no. 7, p. 073 036, 2014.

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