Role of Coherence and Degeneracies in Quantum Synchronisation

Parvinder Solanki[1], Noufal Jaseem[1], Michal Hajdušek[2] & Sai Vinjanampathy[1,3]

[1]Department of Physics, Indian Institute of Technology-Bombay, Powai, Mumbai 400076, India.
[2]Keio University Shonan Fujisawa Campus, 5322 Endo, Fujisawa, Kanagawa 252-0882, Japan.
[3]Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore.

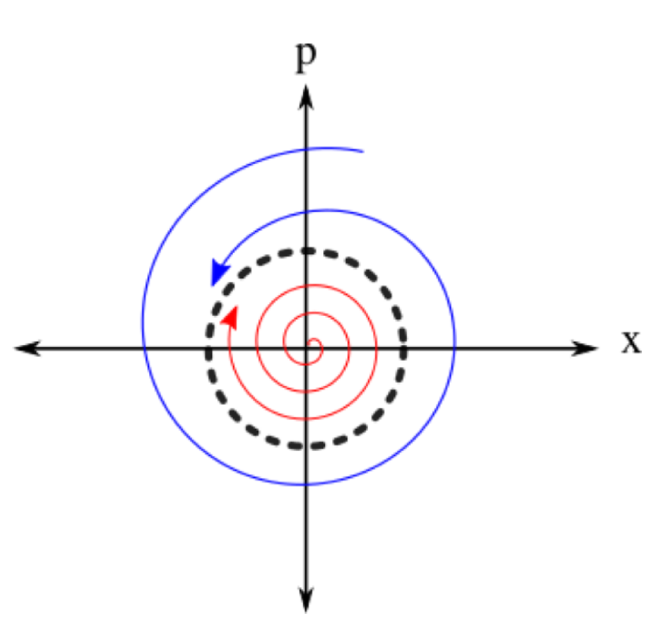
Objectives

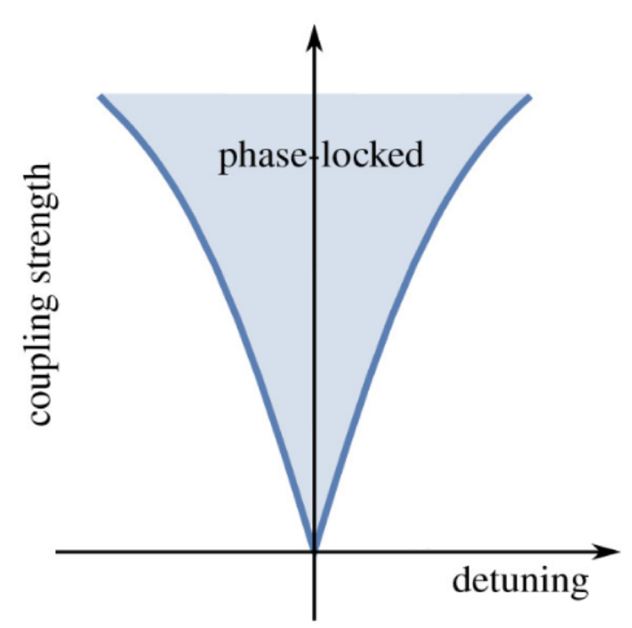
- Coherent output of the heat engines/lasers have been related to the coherences present in the system.
- Coherences also implies the steady state synchronisation in quantum limit cycle oscillators.
- Recently, relationship between the power output and steady state synchronisation of Scovil and Schulz-Dubois heat engine was established[1] which we discuss here in detail.
- In our work, we investigate the change in relationship between the synchronisation and thermodynamic quantities for two mutually **coupled thermal machines**.

What is synchronisation?

"Synchronisation is the adjustment of rhythms of weakly coupled self sustained oscillators".

- Basic features of self sustained oscillators are:
  1. Dissipation, power source and non-linearity.
  2. Form of oscillation is independent of initial position.
  3. Oscillations are stable to (at least rather small) perturbations.





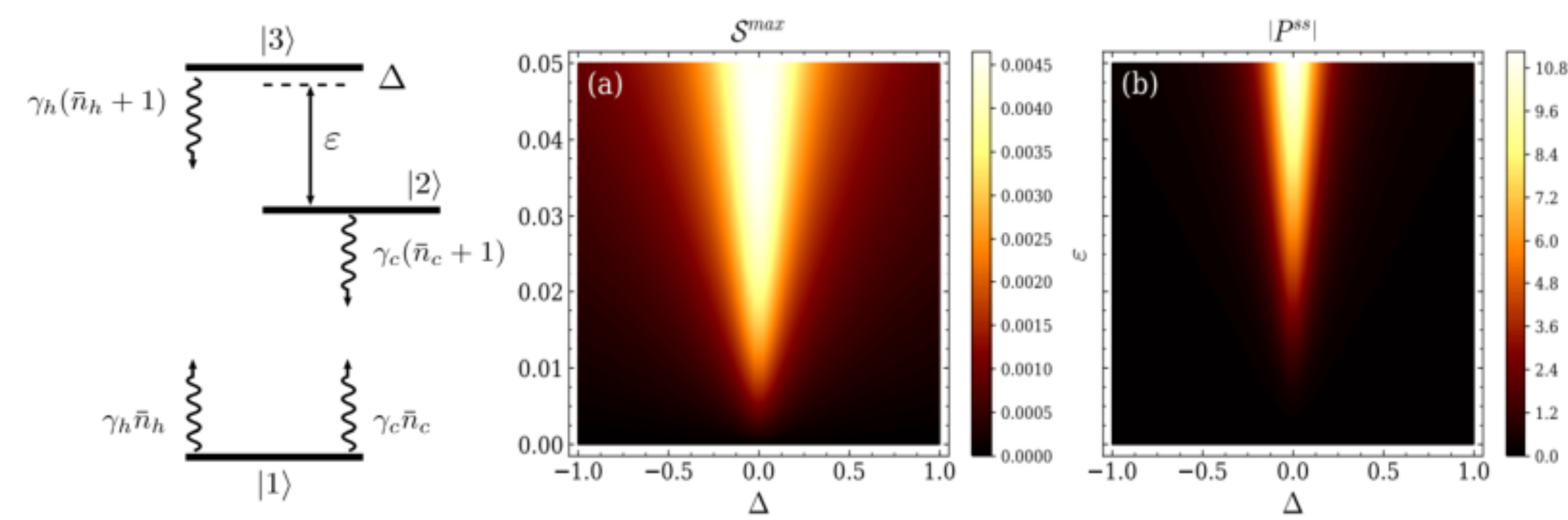
Steps to study sync

1. Establish a valid **limit cycle**.
2. Apply a perturbation: **drive/coupling**.
3. A valid **synchronisation measure** to account for phase locking.

Sync in Scovil and Schulz-Dubois Heat Engine[1]

Establishing a valid **limit cycle**:

- Coherent state: $|n_3\rangle = |\theta, \zeta, \phi_1, \phi_2\rangle = |cos(\theta), e^{i\phi_1} cos\zeta sin\theta, e^{i\phi_2} sin\zeta sin\theta\rangle$.
- $e^{iH_0t}|\theta, \zeta, \phi_1, \phi_2\rangle = |\theta, \zeta, \phi_1 - \omega_{21}t, \phi_2 - \omega_{31}t\rangle$; $H_0 = \Sigma_i \omega_i |i\rangle\langle i|$.
- ϕ_i's are free phases which can be locked on by external drive/other oscillator.
- Master eqn.: $\dot{\rho}^R = -i[H_0^R + V, \rho^R] + \mathcal{D}_{13}^h[\rho] + \mathcal{D}_{12}^c[\rho^R]$; $H_0^R = \Delta\sigma_{33}, V = \epsilon S_x$.
- $Q(\theta, \zeta, \phi_1, \phi_2) = \langle n_3|\rho|n_3\rangle$ shows equiprobable distribution of phases ϕ_i's for the steady state in absence of drive ($\varepsilon = 0$) representing limit cycle behaviour.



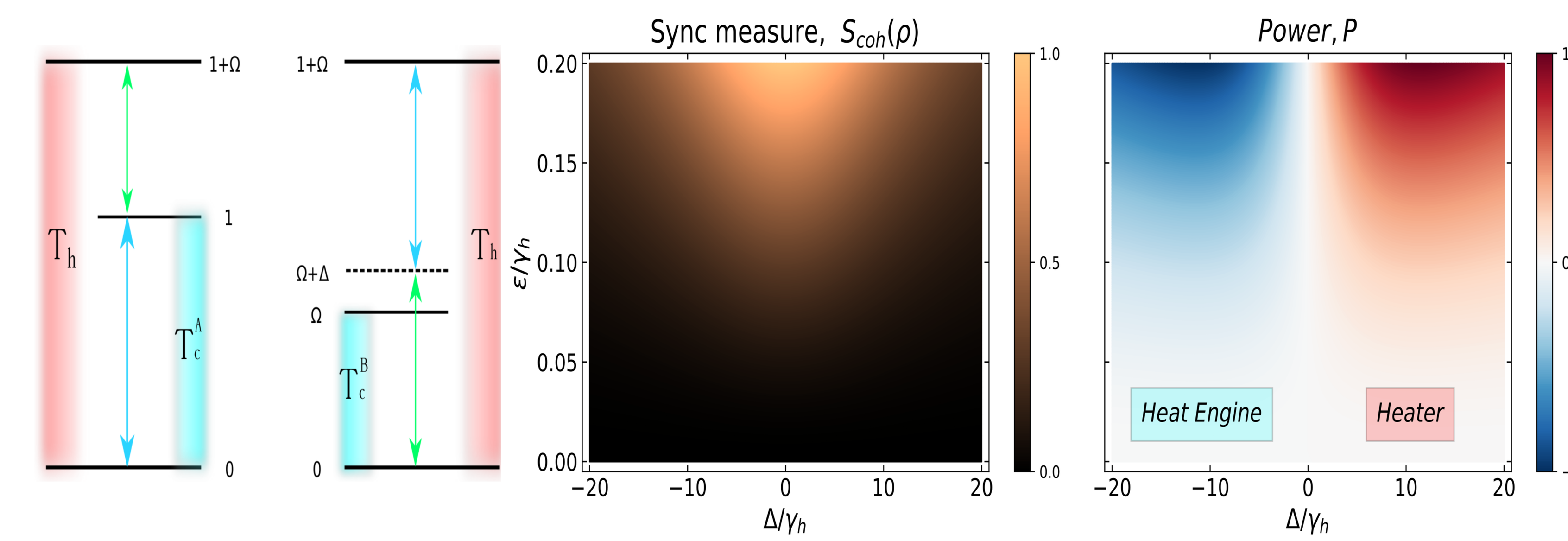
- Perturbation: External drive, $\varepsilon \neq 0 \rightarrow$ phases ϕ_i's can be localized.
- **Synchronisation measure**: $S(\phi_1, \phi_2) = \int d\Omega Q(\theta, \zeta, \phi_1, \phi_2) - \frac{1}{4\pi^2}$;
- $\dot{E} = Tr\{H_0^R \dot{\rho}^R\} = \underbrace{-iTr\{[H_0^R + V, \rho^R]H_0^R\}}_{Power} + \underbrace{Tr\{\sum_i \mathcal{D}_i[\rho]H_0^R\}}_{Heat}$.
- Steady state power: $P_{ss} = -i\mathrm{Tr}([H_0^R + V, \rho_{ss}]H_0^R) = 2\epsilon\omega_{32}\mathrm{Im}[\rho_{23}^{ss}]$.

Validity of 'local' master equation

- 'Global' master equation produces thermodynamically consistent results but require a clean timescale separation which might break down in presence of degeneracies.
- 'Local dissipators' are found to coincides with the zeroth order of the Redfield dissipators if degeneracy of the bare Hamiltonian H_0 changes after turning on the interaction term V.

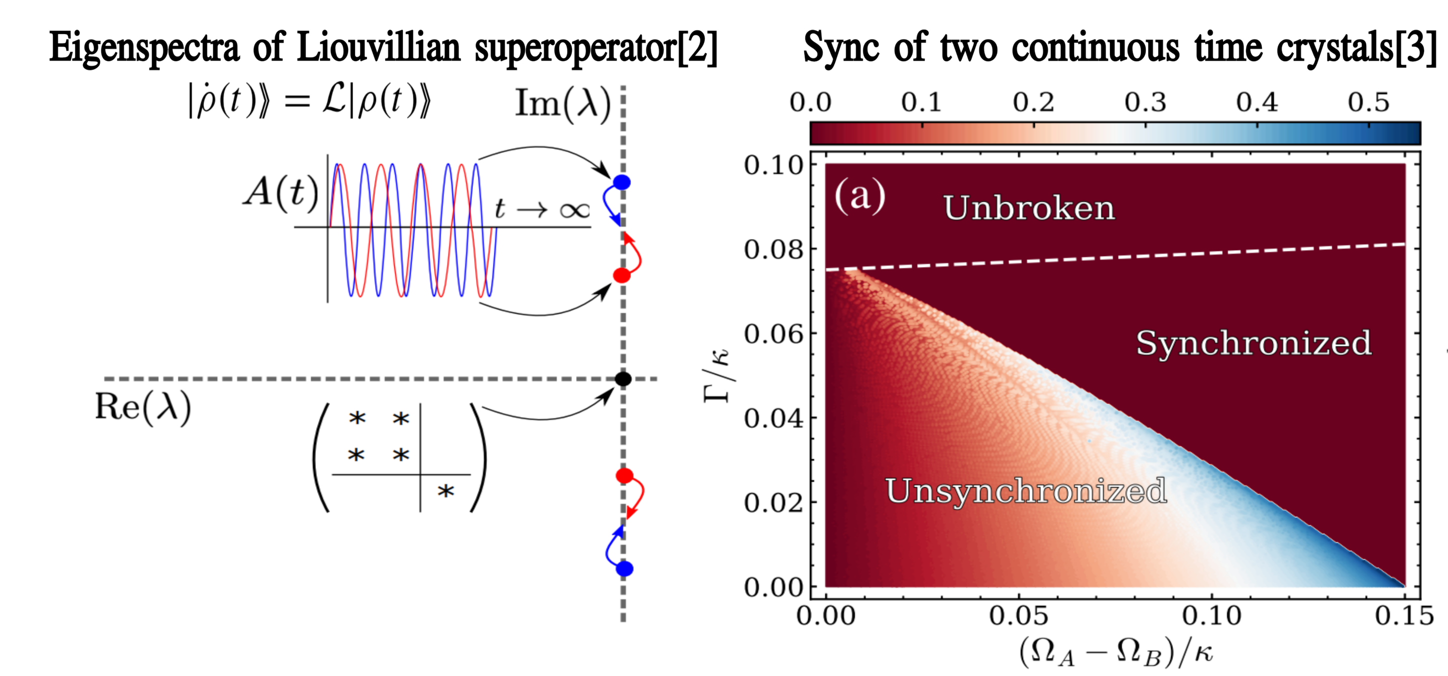
Email: psolanki@phy.iitb.ac.in

Two coupled thermal machine!![2]



- $H_0 = \sigma_{22}^A + (1+\Omega)\sigma_{33}^A + (\Omega+\Delta)\sigma_{22}^B + (1+\Omega)\sigma_{33}^B$.
- $V = \varepsilon(\sigma_{23}^A\sigma_{21}^B + \sigma_{12}^A\sigma_{32}^B + h.c.)$.
- For $\Delta = 0$ interaction is energy conserving, i.e. $[H_0, V = 0]$.
- Relative entropy of synchronisation: $S_{coh}(\rho) = S(\rho_{diag}) - S(\rho)$
- $P = -i\mathrm{Tr}([H_0 + V, \rho]H_0) = 2g\Delta\ \mathrm{Im}[\rho_{35}^{ss} + \rho_{75}^{ss}]$; $\rho^{ss} = \Sigma_{i,j=1}^{9} \rho_{ij}^{ss}|i\rangle\langle j|$.

A different variety of sync[2, 3]



References

[1] Noufal Jaseem, Michal Hajdušek, Vlatko Vedral, Rosario Fazio, Leong-Chuan Kwek, and Sai Vinjanampathy. *Phys. Rev. E*, 101:020201, Feb 2020.

[2] Parvinder Solanki, Noufal Jaseem, Michal Hajdušek, and Sai Vinjanampathy. *Physical Review A*, 105(2):L020401, 2022.

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