A new thermodynamic measure of entanglement

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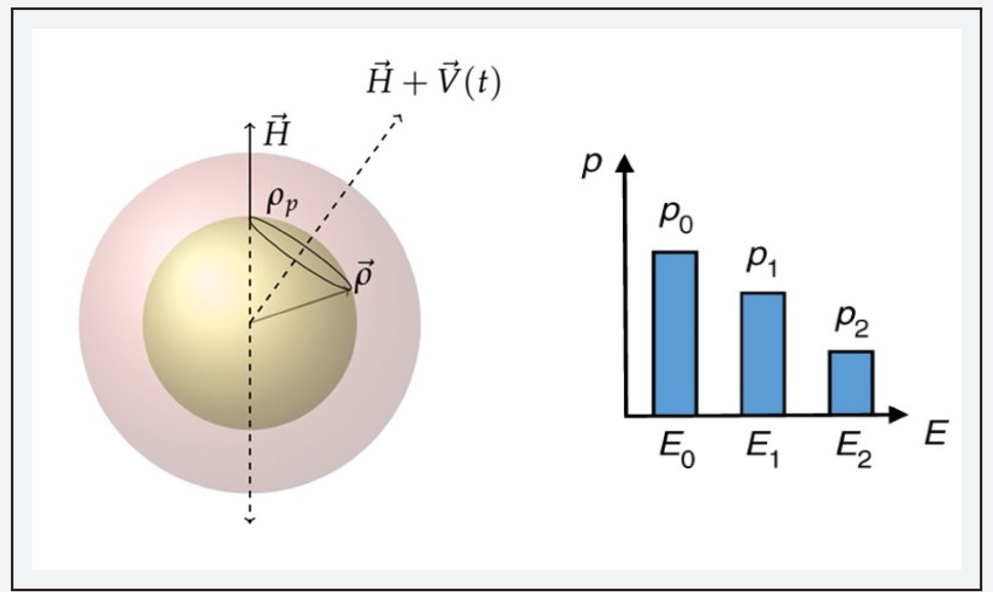
Maximum extractable work from a closed system: concept of Ergotropy

Maximum extractable work from a closed system ρ_s is defined by:

$$W_e(\rho_s) = (\rho_s H_s) - \min_U (U[\tau]\rho U[\tau]^\dagger H_s).$$

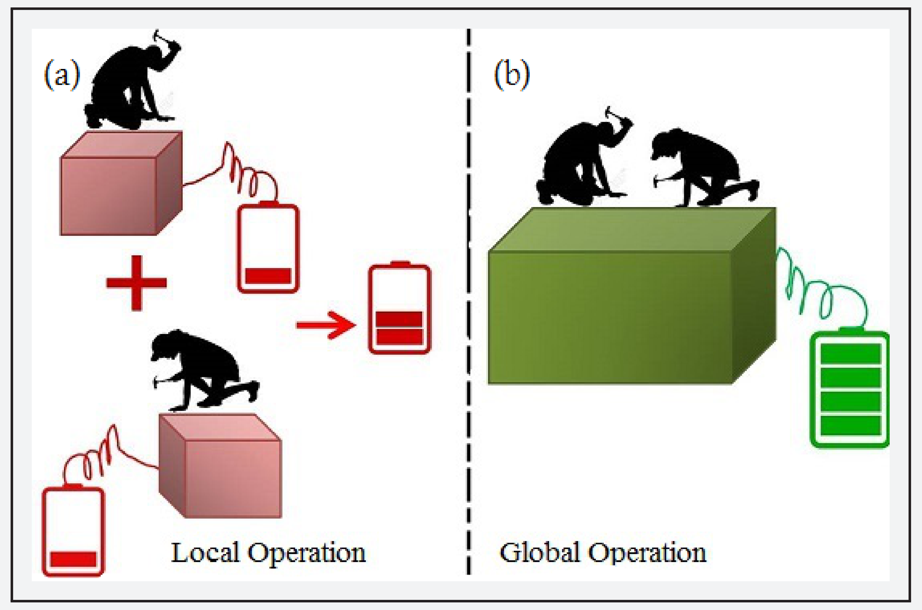
Where, $U[\tau] = \overleftarrow{\exp}\left(-i\hbar \int_0^\tau dt\,(H_s + V(t))\right)$, the time-dependent potential $V(t)$ starts at $t=0$ and decouples from the system at $t=\tau$. Under this dynamics final state would be passive state.



$$W_e(\rho_s) = (\rho_s H_s) - (\rho_s^p H_s).$$

If $H_s = \sum_i E_i |i\rangle\langle i|$, then $\rho_s^p = \sum_i p_i |i\rangle\langle i|$, where $E_i \le E_j$ and $p_i \ge p_j$.

Ergotropic gap



$$W_e^g = \mathrm{Tr}(\rho_{AB}H_g) - \min_{U\in\mathcal{L}(\mathcal{H}_A\otimes\mathcal{H}_B)} \mathrm{Tr}\{U\rho_{AB}U^\dagger H_g\}$$
$$= \mathrm{Tr}(\rho_{AB}H_g) - \mathrm{Tr}(\rho_{AB}^p H_g)$$

Ergotropic gap

Here, $H_g = H_A \otimes I_B + I_A \otimes H_B$.
$W_e^x := \mathrm{Tr}(\rho_X H_X) - \mathrm{Tr}(\rho_X^p H_X)$

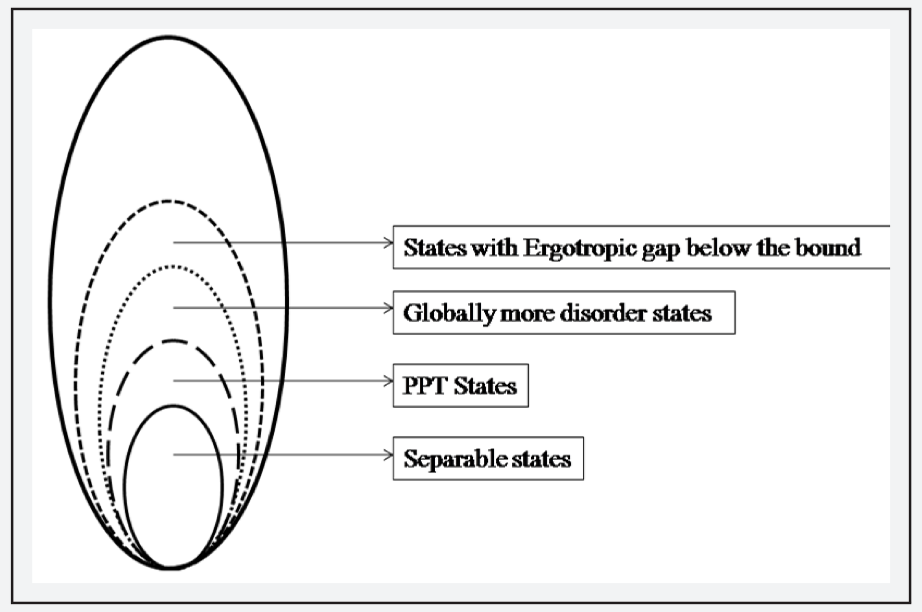
$$\Delta_{A|B} = W_e^g - \{W_e^A + W_e^B\}$$
$$= \mathrm{Tr}(\rho_A^p H_A) + \mathrm{Tr}(\rho_A^p H_A) - \mathrm{Tr}(\rho_{AB}^p H_g)$$

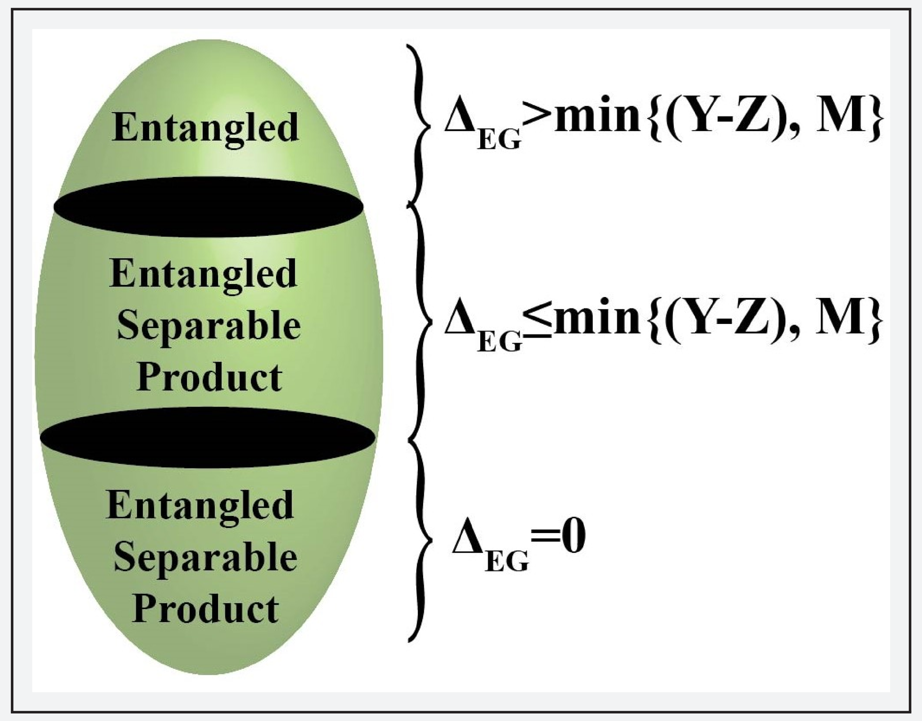
Main results

Proposition: A multipartite pure state governed by the general Hamiltonian is entangled if and only if it has non-zero ergotropic gap.

Theorem 1: Ergotropic gap of a pure bipartite state $|\phi\rangle^{AB}$ is greater or equal to that of $|\psi\rangle^{AB}$ if $\lambda(|\phi\rangle) \prec \lambda(|\psi\rangle)$, where $\lambda(|\phi\rangle)$ and $\lambda(|\psi\rangle)$ correspond to the spectrum of the individual marginals.

Theorem 2:





Applications

- A two-qubit state with maximally disordered marginals can be necessarily and sufficiently characterized by our criteria.
- For the case of a pure two-qubit system, EG becomes an entanglement measure which is robust in nature.
- Recently, we have generalized this measure to capture the genuine entanglement.

Passive state energy is a new entanglement monotone

Define a real function $f: \mathscr{D}(\mathscr{H}_A^d) \to \mathbb{R}$, by

$$f(\rho_A) = \mathscr{E}(\psi_{AB})$$

(i) *Unitary invariant:* $f(\rho) = f(U\rho U^\dagger)$.
(ii) *Concavity:* $f(\sum_i \lambda_i \rho_i) \ge \sum_i \lambda_i f(\rho_i)$

- Passive state energy follows the above two criterion. More explicitly we can show that: if ψ_{AB} to ϕ_{AB} transformation is possible under LOCC then $\mathrm{Tr}(\rho_A^p(\psi)H_A) \le \mathrm{Tr}(\rho_A^p(\phi)H_A)$.
- It is an independent thermodynamic measure than entropy and only measure which build a direction connection between thermodynamics and entanglement theory.

References

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