Measurement-Induced Manipulation of Quantum Coherence

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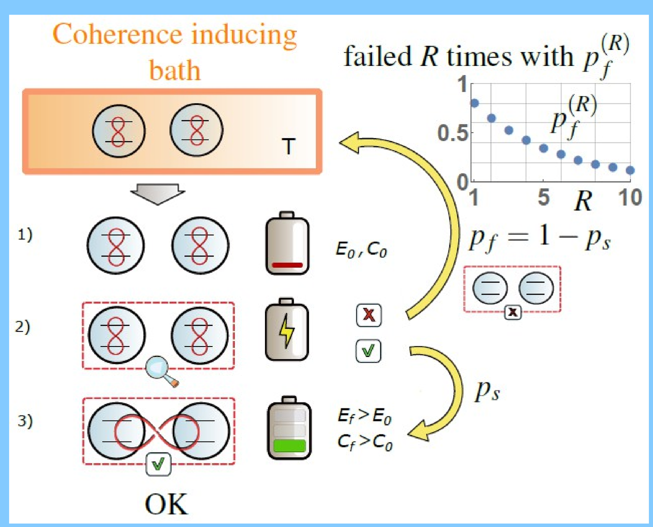
The poster provides an outline of our work on manipulations of different forms of coherence by diagonal (in the basis of incoherent states) quantum measurements. In [1] we have proposed a measurement-based protocol for (energy basis) coherence synthesis from individual systems into an increased global coherence of the compound system. The building block of our system is a pair of independent and non-interacting copies of two-level system (TLS) in an initial state with low excited state population and low initial coherence. A universal option to achieve effective protocol is the usage of global projector diagonal in the energy basis. Generalization to a certain class of diagonal POVM allows for protocol optimization, e.g. for the output state coherence. The results reveal that unconstrained optimization results in local filters and the optimized coherence appears locally on each TLS. Our work [2] answers the question whether such local filters are optimal as well, if constrained optimization is considered. The answer is negative, as for given protocol success probability the optimal filters are non-local. These results are verified in a proof-of-principle linear-optical experiment. The pairwise diagonal projector used in [1] proves its universality in a more general settings. Generalized protocol [3] uses input of N non-interacting copies of TLS with low initial energy and coherence. Sequential application of the projector synthesizes input states into output with higher coherence increasing with N. We study different quantity called mutual coherence [3] as well. It quantifies coherence of the compound system not present locally in marginal states of subsystems. Being a new quantity, we again study properties of mutual coherence on a paradigmatic example of a pair of TLS. We characterize the states optimizing mutual coherence in this Hilbert space. We specify and experimentally realize [4] filters transforming certain input states into these optimal states.

Main idea and basic results [1]

Projector-based protocol and pure TLS states:



$$\hat{H}_j = \frac{E}{2}\left(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|\right), \quad j = 1,2,$$

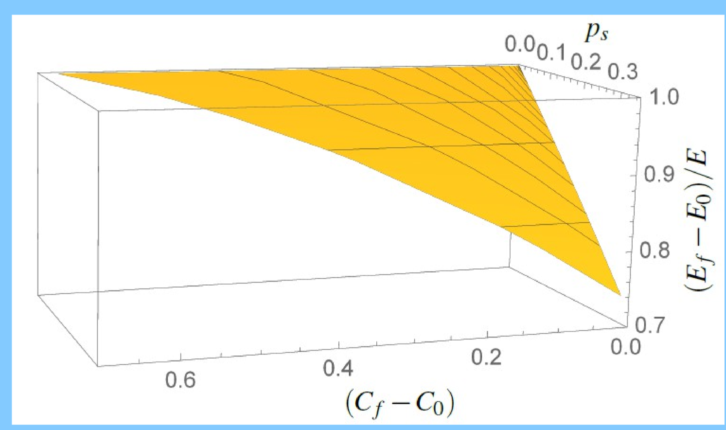
$$C(\hat{\rho}) = S(\hat{\rho}_{diag}) - S(\hat{\rho})$$

$$|\psi_j\rangle = \sqrt{p}\,|e_j\rangle + \sqrt{1-p}\,|g_j\rangle, \quad j = 1,2,$$

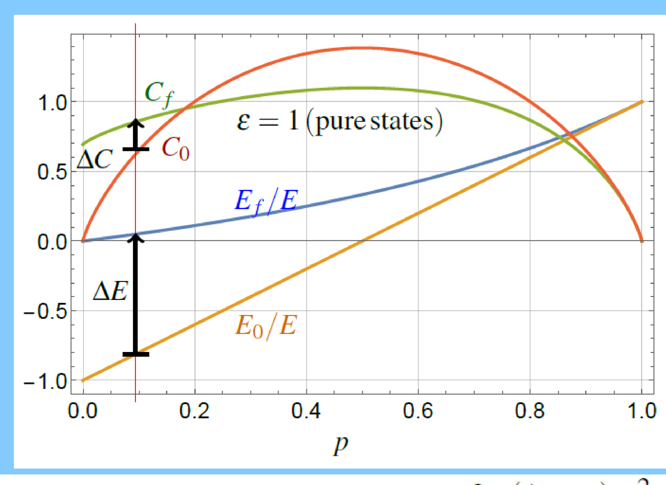
$$\hat{P}_0 = |g_1 g_2\rangle\langle g_1 g_2|, \quad \hat{P}_1 = \hat{1} - \hat{P}_0$$

$$|\Psi_i\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

$$|\Psi_f\rangle = \frac{\hat{P}_1|\Psi_i\rangle}{\sqrt{p_s}} = \frac{p\,|e_1 e_2\rangle + \sqrt{p(1-p)}\,(|e_1 g_2\rangle + |g_1 e_2\rangle)}{\sqrt{p(2-p)}}, \quad p \neq 0,$$



$$C_f \approx \ln 2\left(\frac{2}{5}C_0 + 1\right)$$



$$E_0 = \langle\Psi_i|\hat{H}|\Psi_i\rangle = (2p-1)E, \quad \langle\Delta E_0^2\rangle = 2p(1-p)E^2$$

$$E_f = \langle\Psi_f|\hat{H}|\Psi_f\rangle = \frac{pE}{2-p}, \quad \langle\Delta E_f^2\rangle = \frac{2p(1-p)E^2}{2-p}, \quad p \neq 0$$

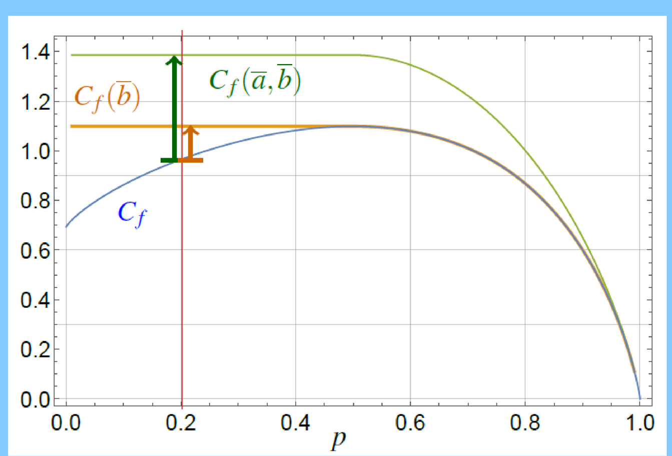
Generalizations

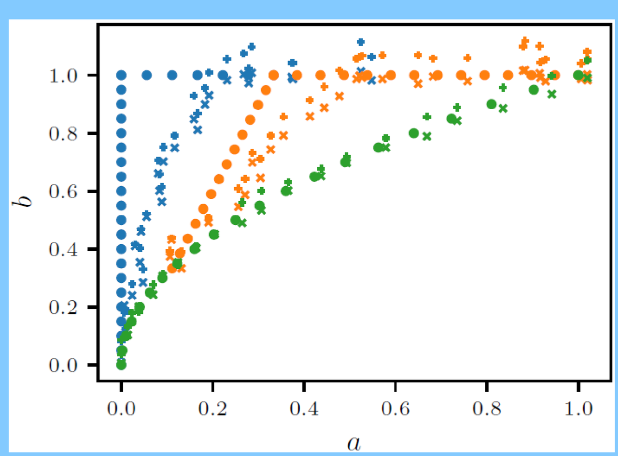
POVM-based protocol:

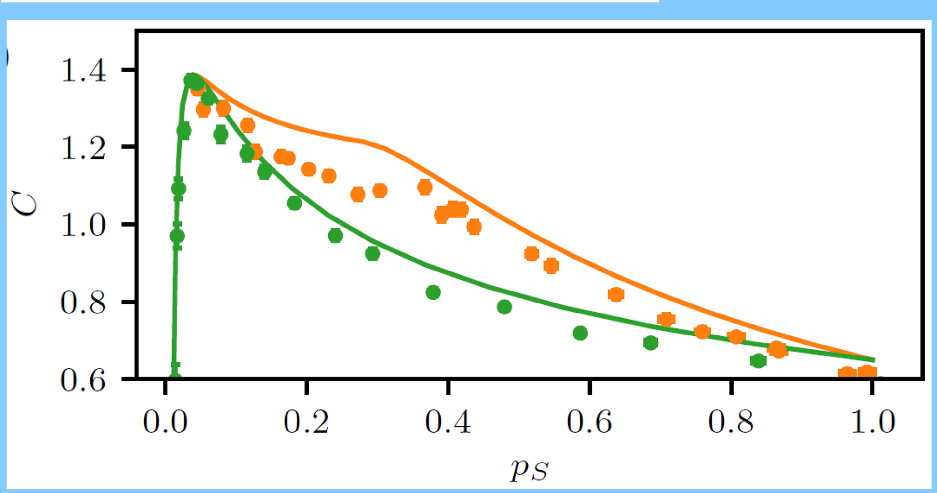
$$\hat{M} = a|00\rangle\langle 00| + b(|01\rangle\langle 01| + |10\rangle\langle 10|) + |11\rangle\langle 11| \quad a \leq b^2$$

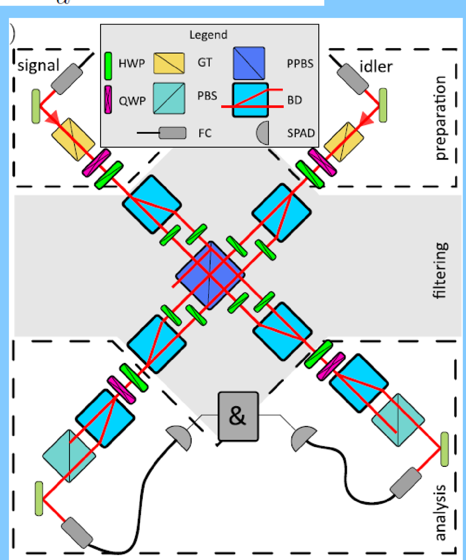
$$\hat{M} = (b|0\rangle\langle 0| + |1\rangle\langle 1|)^{\otimes 2} \quad a = b^2$$

Coherence-optimized POVM element: [2]









N TLS, projector-based protocol: [3]

$$|\Psi_i^{(N)}\rangle = \bigotimes_{j=1}^{N} |\psi_j\rangle$$

Global

$$\hat{P}_0^{(N)} = \bigotimes_{j=1}^{N} |g_j\rangle\langle g_j|, \quad \hat{P}_1^{(N)} = \hat{1} - \hat{P}_0^{(N)}$$

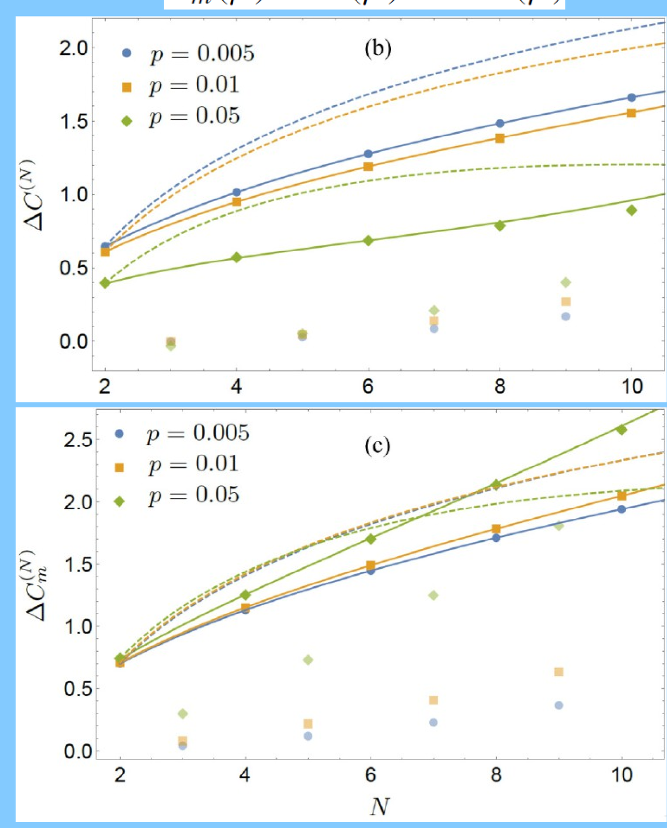
$$p_s^{(N)} = 1 - (1-p)^N$$

$$\Delta C^{(N)} \approx \ln N - \frac{N+1}{2}(1 - \ln p)p$$

$$\Delta C_m^{(N)} \approx \ln N + \frac{N-1}{2}(1 - \ln p)p - \frac{(N-1)^2}{N-2}\ln(N-1)p$$

Mutual coherence:

$$C_m(\hat{\rho}) = C(\hat{\rho}) - C^{\text{loc}}(\hat{\rho})$$



$$\Delta C^{(N)} > \cdots > \Delta C^{(3)} > \Delta C^{(2)} \quad C_f^{(N)\text{loc}} < C_0^{(N)\text{loc}}$$

Pairwise

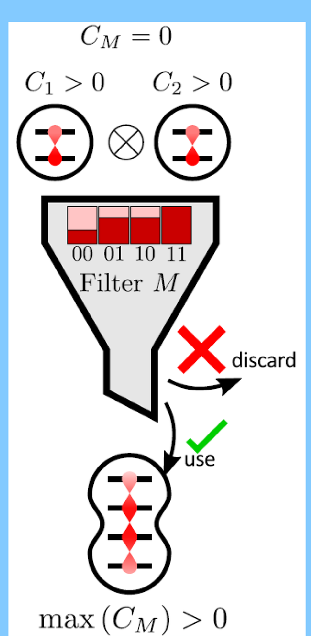
$$\hat{P}_1^{\text{tot}} \equiv \prod_{i=1}^{N-1} \hat{P}_1^{(i-i+1)}, \quad \hat{P}_1^{(i-j)} = \hat{1} - |g_i g_j\rangle\langle g_i g_j|$$

$$p_s^{(N)} \approx \left(\frac{N}{2} + 1\right)p^{\frac{N}{2}}, \quad \text{even } N$$

$$\Delta C^{(N)} \approx \ln\left(\frac{N}{2} + 1\right) - \frac{N(20-N)}{24}(1 - \ln p)p, \quad \text{even } N$$

$$\Delta C_m^{(N)} \approx \ln\left(\frac{N}{2} + 1\right) + \frac{N(N+4)}{24}(1 - \ln p)p - \ln\left[\frac{3}{2}\left(\frac{N}{2} + 1\right)!\right]p, \quad \text{even } N$$

Mutual coherence C_M for a pair of qubits [4]



$$|\psi_{\text{in}}\rangle = \left(\sqrt{p}\,|1\rangle + \sqrt{1-p}\,|0\rangle\right)^{\otimes 2}$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\psi_4\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$

$$C_M(\psi_2) = 1 \text{ and } C_M(\psi_4) = 2$$

3D subspace:

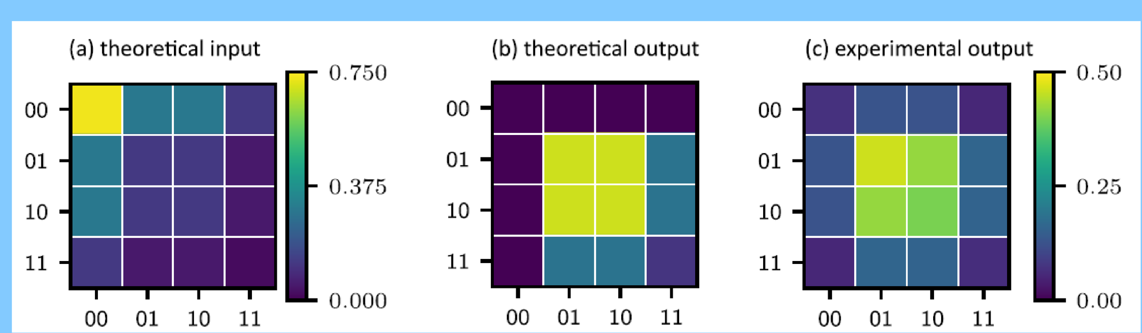
$$|\psi_3\rangle = c|11\rangle + \sqrt{\frac{1-c^2}{2}}(|01\rangle + |10\rangle)$$

$$c \approx 0.277 \quad \sqrt{\frac{1-c^2}{2}} \approx 0.679$$

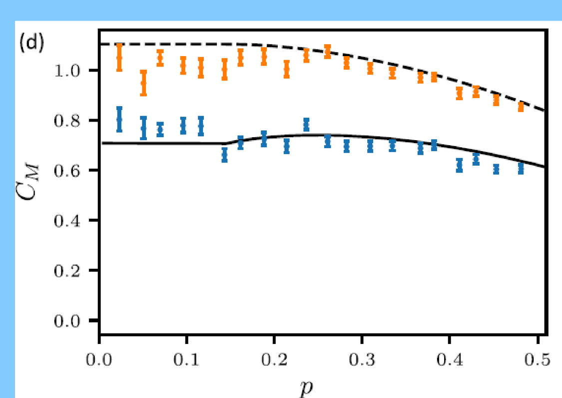
$$C_M(\psi_3) \approx 1.1$$

$$\hat{M} = q(|01\rangle\langle 01| + |10\rangle\langle 10|) + |11\rangle\langle 11|, \; p \leq p_{\text{th}} = 2c^2/(1+c^2)$$

$$q = \sqrt{p(1-p_{\text{th}})/[p_{\text{th}}(1-p)]}$$



$$\hat{M}: \quad D = 3 \quad p = 0.125 \quad F = 0.914 \quad C_M = 0.78$$



Conclusions:

- ***E* and *C* can be conditionally increased via <u>projection</u> incoherent in energy basis, synthesizing a pair of TLS into a larger coherent battery**
- **use of <u>POVM</u> allows for coherence optimization, reaching its maximum for separable filters; with constraints nonseparable filters win**
- **generalization for *N* TLS is possible in several ways, e.g., via global or pairwise protocol, with qualitatively comparable results, protocols increase coherence and mutual coherence with number *N* of TLS**
- **maximum mutual coherence C_M states of diferent dimension reveal nontrivial structure even for the simplest Hilbert space of two TLS**

Ph.D. students wanted: email us!

References: [1] M. Gumberidze, M. Kolář, and R. Filip, Scientific Reports 9, 19628 (2019).

[2] R. Stárek, M. Mičuda, M. Kolář, R. Filip, and J. Fiurášek, Quantum Sci. Technol. 6, 045010 (2021).

[3] M. Gumberidze, M. Kolář, and R. Filip, Phys. Rev. A, 105, 012401 (2022).

[4] N. Horová, R. Stárek, M. Mičuda, J. Fiurášek, M. Kolář, and R. Filip, submitted.

The support of the projects of the Grant Agency of the Czech Republic (GAČR) No. 20-16577S, 951737 NONGAUSS (under H2020-EU.4.b) is acknowledged.