Stochastic entropy production for continuous measurements of an open quantum system

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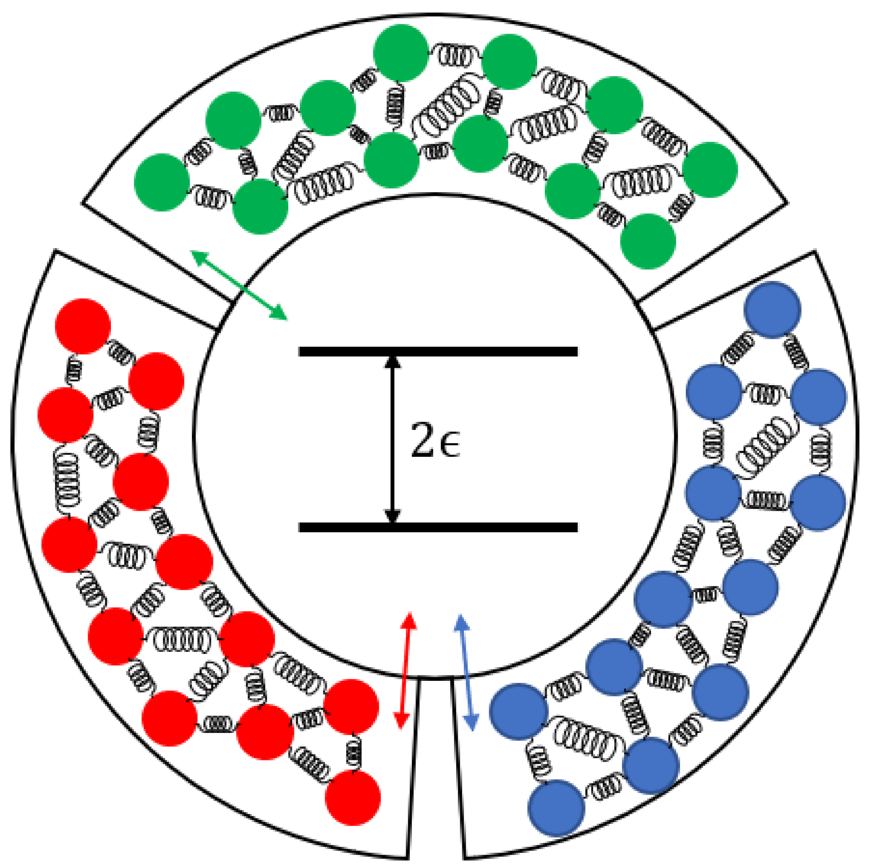
The message

- It is suggested that the reduced density matrix of an open quantum system evolves continuously and pseudorandomly, like a classical Brownian particle.
- Hamiltonian coupling to the environment can thermalise the system and/or 'measure' observables.
- Dynamics are described using Quantum State Diffusion and time-dependent environmental coupling strengths.
- Stochastic entropy production is associated with the 'selection' of an eigenstate of an observable.
- This represents (in part) a change in the uncertainty of the reduced density matrix over its continuum of possibilities.
- The mean stochastic entropy production satisfies a second law and differs from von Neumann entropy.

Two level system coupled to THREE harmonic baths

- Each bath is coupled to the system through a Pauli matrix σ_k multiplied by a strength of interaction.
- System Hamiltonian $H_{sys} = \epsilon\sigma_z$.



Dynamical framework

Stochastic Lindblad equation for the reduced density matrix [1]

$$d\rho = -i\left[H_{sys},\rho\right]dt + \sum_k \left[\left(L_k\rho L_k^\dagger - \frac{1}{2}\left\{L_k^\dagger L_k,\rho\right\}\right)dt + \left(\rho L_k^\dagger + L_k\rho - \mathrm{Tr}\left[\rho\left(L_k + L_k^\dagger\right)\right]\rho\right)dW_k\right]$$

Lindblads valid for **high** temperatures:

$$L_x = \lambda\left(\sigma_x - i\frac{\beta\epsilon}{2}\sigma_y\right)$$

$$L_y = \lambda\left(\sigma_y + i\frac{\beta\epsilon}{2}\sigma_x\right)$$

$$L_z = \lambda\gamma\sigma_z$$

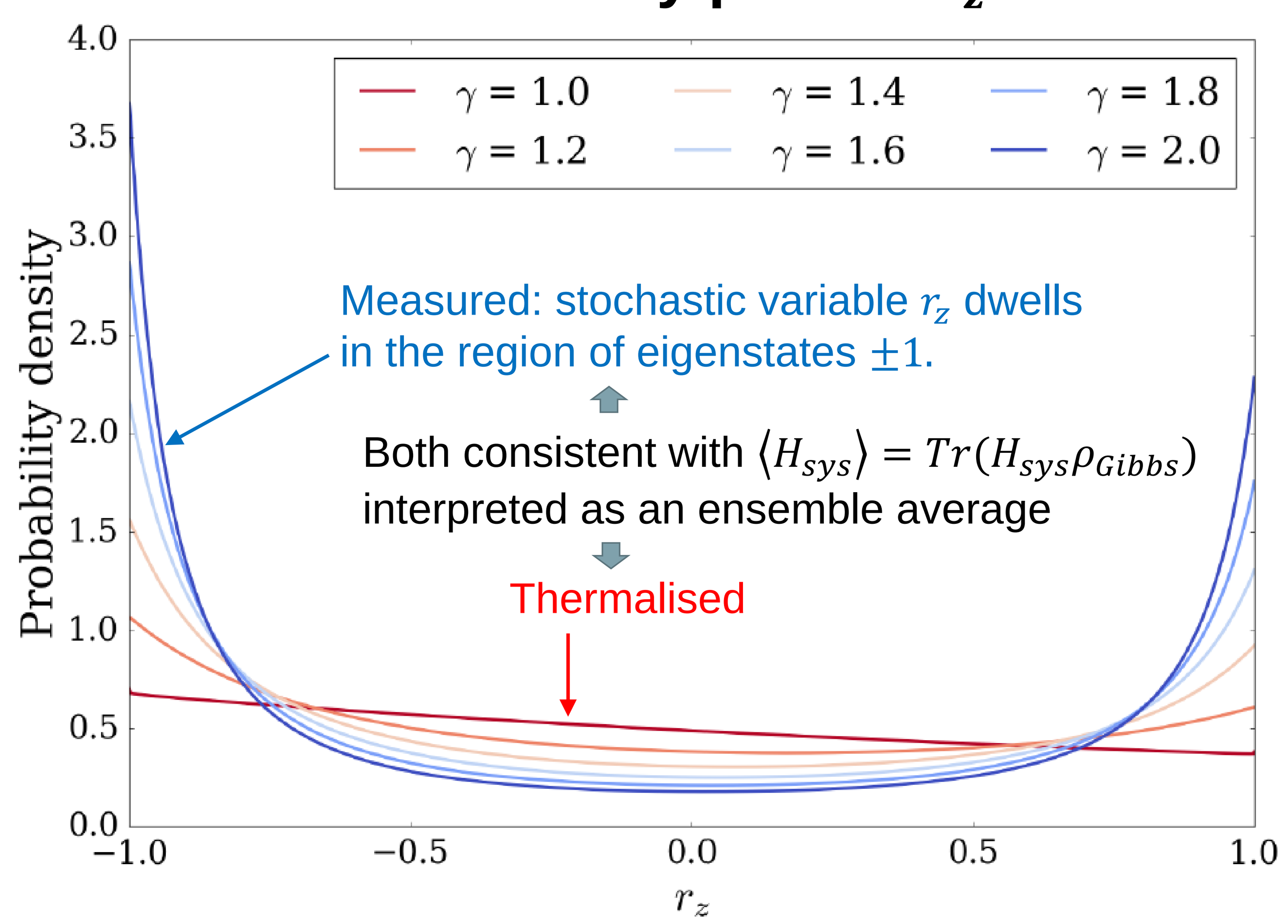
λ: environmental variable
β: inverse temperature

γ = 1: thermalisation
$\gamma > 1$: measurement of H_{sys}

Fokker-Planck eqn for pdf of $r_z = Tr(\rho\sigma_z) = \langle H_{sys}/\epsilon\rangle$:

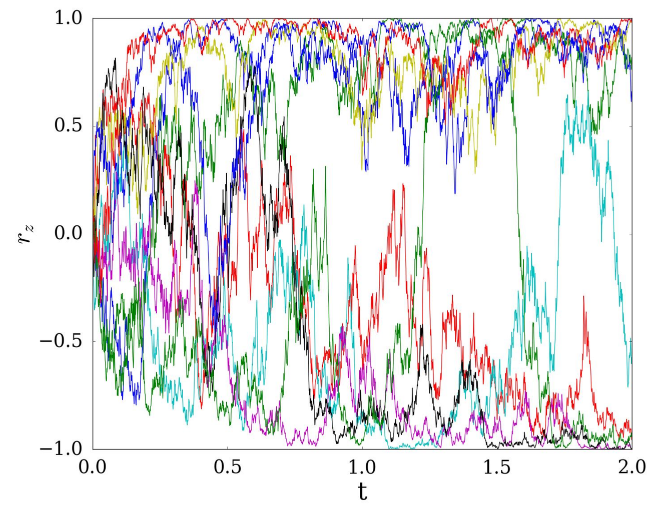
$$\frac{\partial}{\partial t}p(r_z,t) = \frac{\partial}{\partial r_z}\left[\frac{\lambda^2}{2}\left(-2\beta^2\epsilon^2 r_z + 12\beta\epsilon\left(1-r_z^2\right) + 16(1-\gamma^2)r_z\left(1-r_z^2\right)\right)p(r_z,t) + 2\lambda^2\left(1-r_z^2\right)\left(\left(\frac{\beta\epsilon}{2}\right)^2 + \beta\epsilon r_z + (1-\gamma^2)r_z^2 + \gamma^2\right)\frac{\partial}{\partial r_z}p(r_z,t)\right]$$

Stationary pdfs of r_z



Dynamics and thermodynamics of measurement

Measurement of H_{sys} is driven by raising γ above 1.



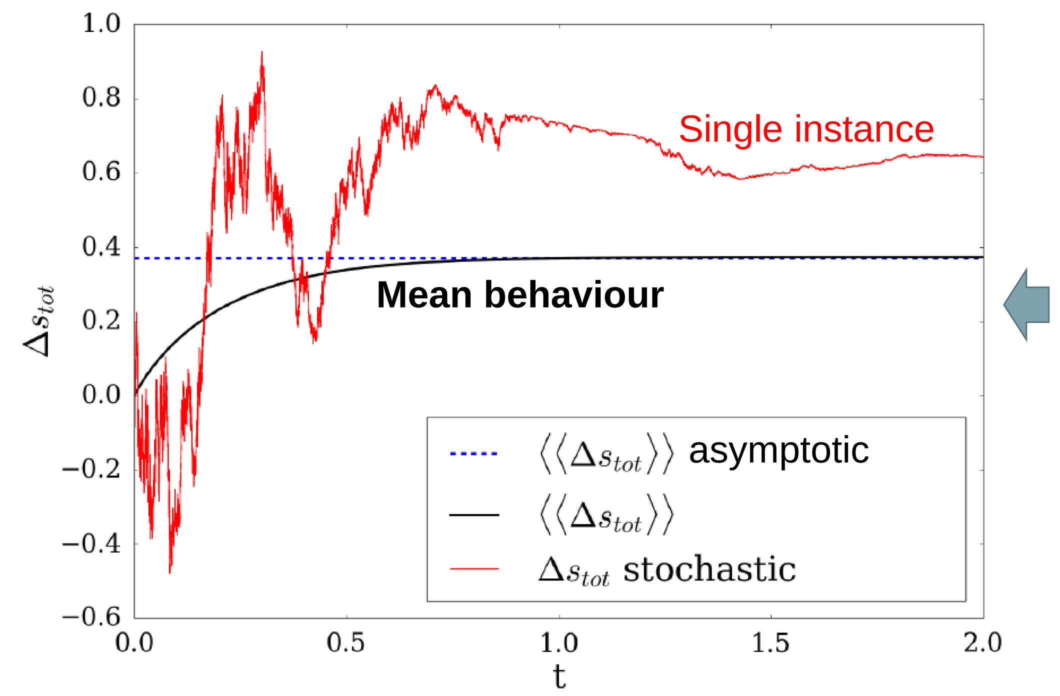
Ensemble of open quantum systems under measurement with $\gamma = 2$.

r_z is drawn stochastically towards eigenstates.

Evolution of stochastic entropy production defined by

$$\Delta s_{tot} = \ln\left(\frac{Prob(\text{forward trajectory})}{Prob(\text{backward trajectory})}\right)$$

follows from the stochastic dynamics of r_z [2,3,4]:



Mean stochastic entropy production increases monotonically.

Individual trajectories fluctuate and give a range of values of total production.

What does it all mean?

- Positive mean stochastic entropy production of measurement is not the same as a change in von Neumann entropy.
- It is the growth in subjective uncertainty in the quantum state of **the system and its environment**, arising from underspecified dynamics and initial conditions.
- The reduced density matrix, and system properties, become less uncertain as a result of measurement but uncertainty propagates into the environment via the coupling.
- Irreversibility in quantum thermodynamics (the growth of uncertainty) is hence very reminiscent of classical situations [5].

References:

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