Fluctuation in Quantum Critical Heat Engine

Rishav Sagar , Victor Mukherjee

Department of Physical Sciences, Indian Institute of Science Education and Research, Berhampur

Abstract

Small thermal quantum machines and systems are typical subjects to non-negligible fluctuations, which can be both of thermal and quantum nature. It no longer suffices to study the mean values but one has to take their fluctuations into account. Characterizing fluctuations in the output of thermal machines is an important requirement in quantum technology. **In this study, we shed some new light on the universal behavior of fluctuations in a many-body quantum Otto cycle in which the working medium is driven across quantum critical points during the unitary strokes.** Under general conditions, we show that fluctuations in output work follow the Kibble-Zurek mechanism,i.e, it follows universal power-law scaling with the driving speed through the critical points.

Preliminaries

Two-point measurement:

Energy determined by projective measurements

Quantum distributions of work and heat:

$$P\,(\text{W or Q}) = \sum_{m,n} \delta\left(\text{W or Q} - (\epsilon_m^\tau - \epsilon_n^0)\right) p_{n\to m}^\tau p_n^0$$

Joint work-heat statistics for one quantum Otto cycle[1]:

$$P\left(W,Q_h\right) = \sum_{i,j,m,n} \delta\left(W - \left(\epsilon_j^0 - \epsilon_i^\tau + \epsilon_m^\tau - \epsilon_n^0\right)\right)$$
$$\delta\left(Q_h - (\epsilon_i^\tau - \epsilon_m^\tau)\right) p_{n\to m}^\tau p_{i\to j}^\tau e^{-\beta_c \epsilon_n^0} e^{-\beta_h \epsilon_i^\tau} / \left(Z_\tau Z_0\right)$$

Characteristic function:

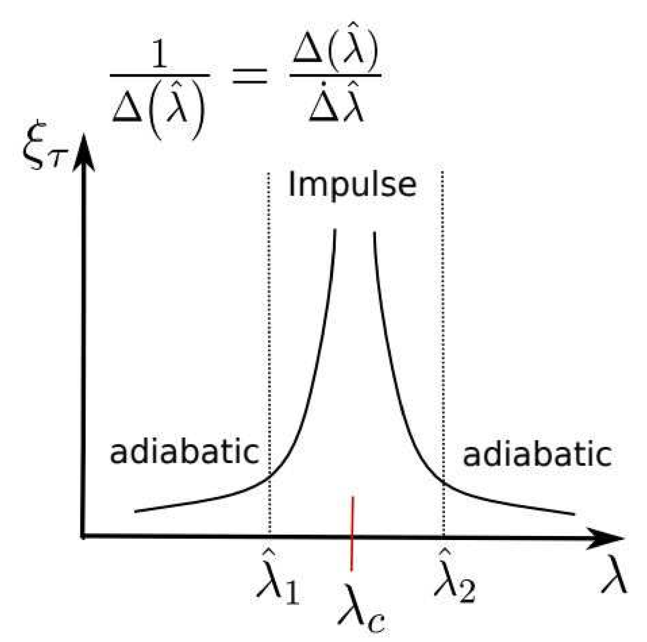
$$\chi\left(\gamma_1,\gamma_2\right) = \iint_{-\infty}^{\infty} dW dQ_h e^{i\gamma_1 W} e^{i\gamma_2 Q_h} P\left(W,Q_h\right)$$

Moments:

$$\left\langle W^m Q_h^n \right\rangle = \frac{\partial^m}{\partial\left(i\gamma_1\right)^m} \frac{\partial^n}{\partial\left(i\gamma_2\right)^n} \chi\left(\gamma_1,\gamma_2\right)\Big|_{\gamma_i=0}$$

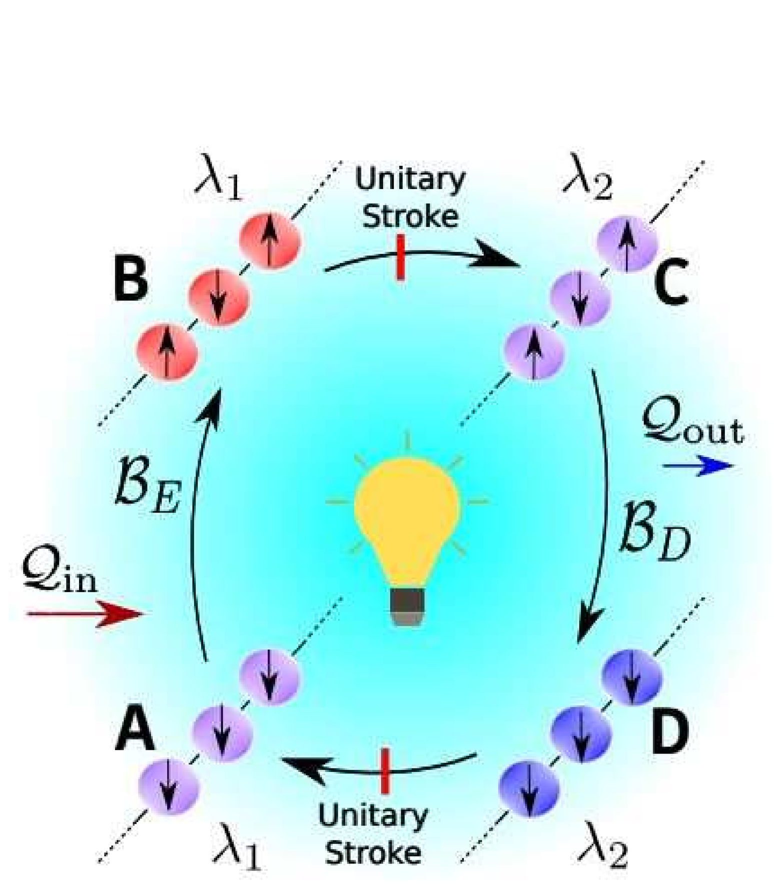
Kibble Zurek Scaling

Adiabatic Impulse Approximation : A system evolves adiabatically away from critical point, while non-adiabatic excitations result close to criticality.

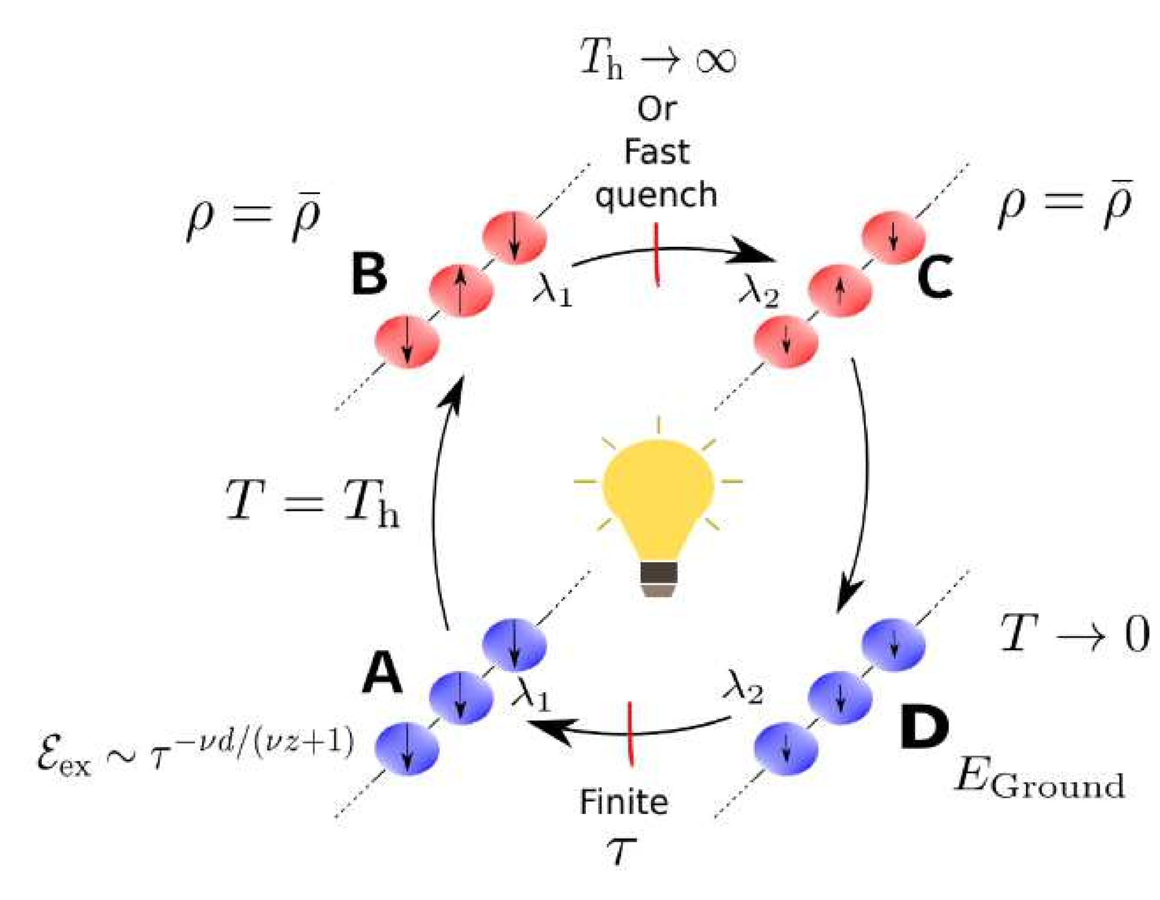


- Energy gap Δ vanishes at the quantum critical point $\lambda = \lambda_c$
- Two competing time scales: Inherent time scale $1/\Delta$
- Time scale associated with the change of hamiltonian $\Delta/\dot{\Delta}$.
- $\lambda(t) = \lambda_c + t/\tau \implies n_{\text{ex}} \sim \tau^{-\nu d/(\nu z+1)}$
- $\langle E_{ex}\rangle \sim \tau^{-\nu d/(\nu z+1)}$
- Also, $Var(E_{ex}) \sim \tau^{-\nu d/(\nu z+1)}$[2]

Kibble Zurek in Otto Engine

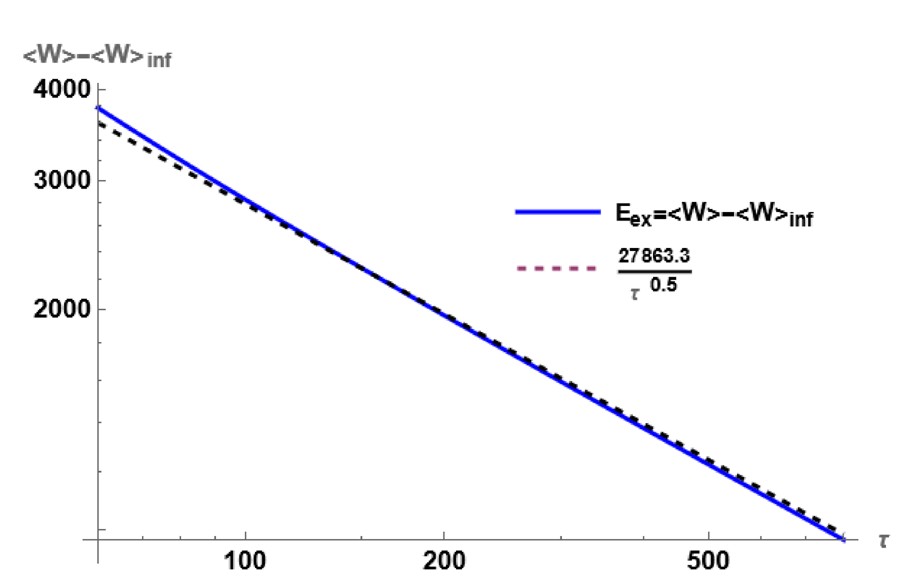


(a) An Otto Cycle

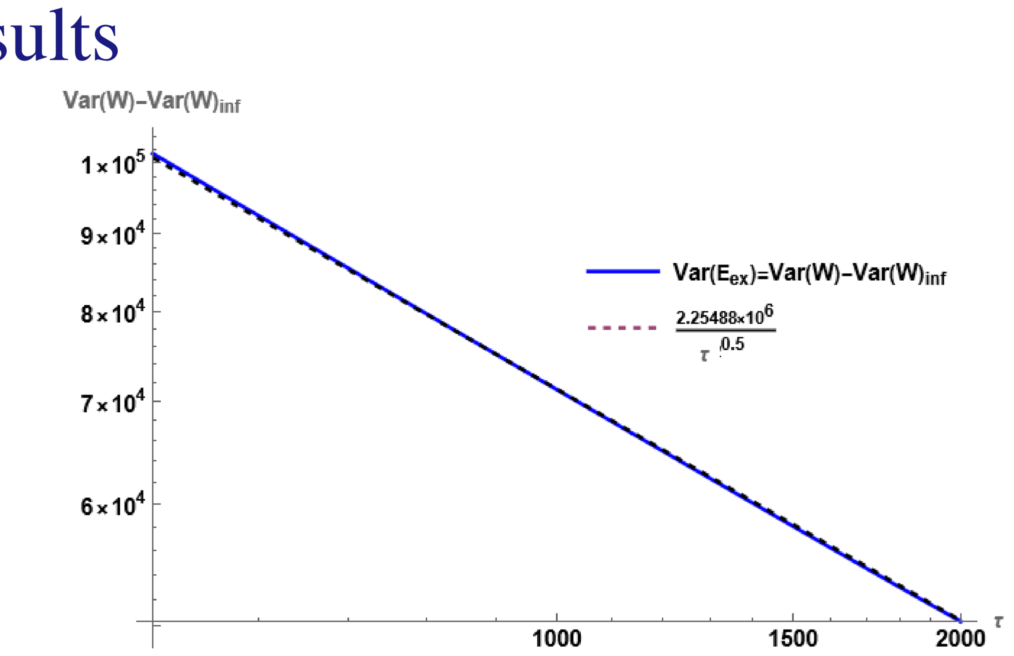


(b) KZM in Otto Cycle[3]

Results



(a) $\langle W\rangle - \langle W_\infty\rangle \sim \tau^{-1/2}$



(b) $\text{Var}(W) - \text{Var}(W)_\infty \sim \tau^{-1/2}$

Universal Scaling of Mean Work

$W = -(Q_{in} + Q_{out})$
Also, $Q_{in} = Q_{in}(\tau\to\infty) - E_{ex}$
$Q_{out} = E_D - E_C = Q_{out}(\tau\to\infty)$

- $\langle W\rangle - \langle W_\infty\rangle \sim \tau^{-\nu d/(\nu z+1)}$

Universal Scaling of Fluctuation

$Var(W) = \sum_k Var(W_k)$
$\because W_k = W_k^\infty + \mathcal{E}(k)_{\text{ex}}$
$\sum_k \text{Var}\left(W_k\right) = \sum_k \text{Var}\left(W_k^\infty\right) + \sum_k \text{Var}\left(\mathcal{E}(k)_{\text{ex}}\right)$

$$\text{Var}(W) = \text{Var}(W)_\infty + \text{Var}\left(\mathcal{E}_{\text{ex}}\right)$$

$$\text{Var}(W) - \text{Var}(W)_\infty \sim \tau^{-\frac{\nu d}{\nu z+1}}$$

Modeling Quantum Engines

- We can use the these universal scalings to choose the best possible materials to design quantum engines.
- For example, working mediums with high dimensions (d) may produce reduced fluctuations in output work: $\text{Var}(W) - \text{Var}(W)_\infty \sim \tau^{-\frac{\nu d}{\nu z+1}}$.

Conclusions

- Existence of Universal Scaling of fluctuations in output work.
- We verified our results by considering transverse-field Ising medium as working fluid in Quantum Otto heat engine.

References

[1] Tobias Denzler and Eric Lutz. Efficiency fluctuations of a quantum heat engine. *Phys. Rev. Research*, 2:032062, Sep 2020.

[2] Zhaoyu Fei, Nahuel Freitas, Vasco Cavina, H. T. Quan, and Massimiliano Esposito. Work statistics across a quantum phase transition. *Phys. Rev. Lett.*, 124:170603, May 2020.

[3] Revathy B. S, Victor Mukherjee, Uma Divakaran, and Adolfo del Campo. Universal finite-time thermodynamics of many-body quantum machines from kibble-zurek scaling. *Physical Review Research*, 2(4), Nov 2020.

Contact information: Rishav Sagar, Department of Physical Sciences, IISER, Berhampur, Odisha, IN-760010 – Phone: +91-8079862823 – Email: *rishav17@iiserbpr.ac.in*;

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