Quantum harmonic oscillator under measurement and feedback

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Abstract

In order to control and manipulate quantum systems it is often necessary to apply measurement and feedback. Recently a Quantum Fokker-Planck Master Equation (QFPME) [1] was derived for the joint distribution of system and detector outcome. Here we apply this formalism to the quantum harmonic oscillator with cooling protocol. We find the optimal ratio of parameters **measurement strength** and **detector lag** to achieve maximum cooling. For x and p measurement we can have ground state cooling for arbitrarily fast protocols. When weakly coupling the system to a thermal bath we can extract power from a single reservoir.

Quantum Fokker-Planck Master Equation[1]

- Weak measurement → white noise
- $\hat{A}$ is the measured operator (Hermitian)
- λ is the measurement strength
- γ is the detector bandwidth (lag)

$$\rho(t+\delta t) = e^{\mathcal{L}} \frac{K(z)\rho K^\dagger(z)}{Tr[K^\dagger(z)K(z)\rho]} \qquad K(z) = \left(\frac{2\lambda\delta t}{\pi}\right)^{1/4} e^{-\lambda\delta(z-A)^2}$$

$$z = \langle A\rangle + \frac{\delta W}{\delta t\sqrt{4\lambda}} \qquad D = \int_{-\infty}^{t} dt' e^{-\gamma(t-t')} z(t')$$

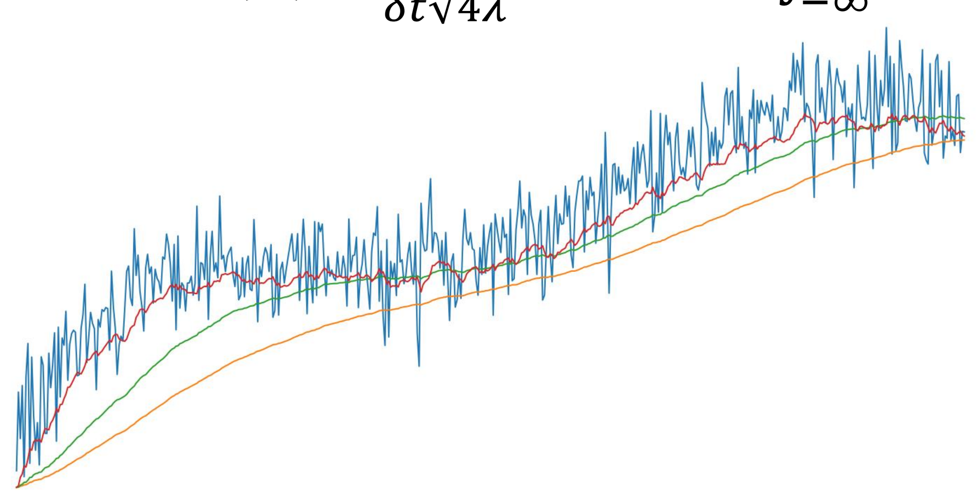

Fig 1.Exponential filtering "lags" behind the signal by averaging past values

$$\boldsymbol{\partial_t\rho(D,t) = \mathcal{L}(D)\rho(D,t) + \lambda\mathcal{D}[A]\rho(D,t) - \frac{\gamma}{2}\partial_D\{A-D,\rho(D,t)\} + \frac{\gamma^2}{8\lambda}\partial_D^2\rho(D,t)}$$

- D is the detector outcome, a proxy for $\langle\hat{A}\rangle$
- $\rho(D,t)$ is the joint distribution for **quantum state** and **detector outcome**

$$\rho(t) = \int dD\, \rho(D,t) \qquad P(D,t) = Tr[\rho(D,t)]$$

- $\mathcal{L}(D)$ is the D-dependent Liouville operator → feedback based on the outcome of detector

If $\gamma \gg \lambda, \omega$ then we have separation of time scales:

$$\partial_t\rho(t) = \left(\mathcal{L}_0 + \frac{1}{\gamma}\mathcal{L}_1\right)\rho(t)$$

X & P-measurement

$$A_x = x \qquad A_p = p \qquad H(D_x,D_p) = \frac{\hbar\omega}{2}\left[(p-D_p)^2 + (x-D_x)^2\right]$$

- Analytical steady state solution

$$\langle H(D)\rangle_S = \frac{\hbar\omega}{2}\left(\frac{\lambda}{\gamma} + \frac{\gamma}{4\lambda}\right) \qquad \sigma_H^2 = \langle H\rangle_S^2 - \left(\frac{\hbar\omega}{2}\right)^2$$

- Optimal condition at $\gamma = 2\lambda$ → Fast energy relaxation time 2γ

$$\langle H(D)\rangle_S = \frac{\hbar\omega}{2} \quad \& \quad \sigma_H^2 = 0 \quad \Rightarrow$$

Trajectory level ground state cooling!

Trajectory simulation

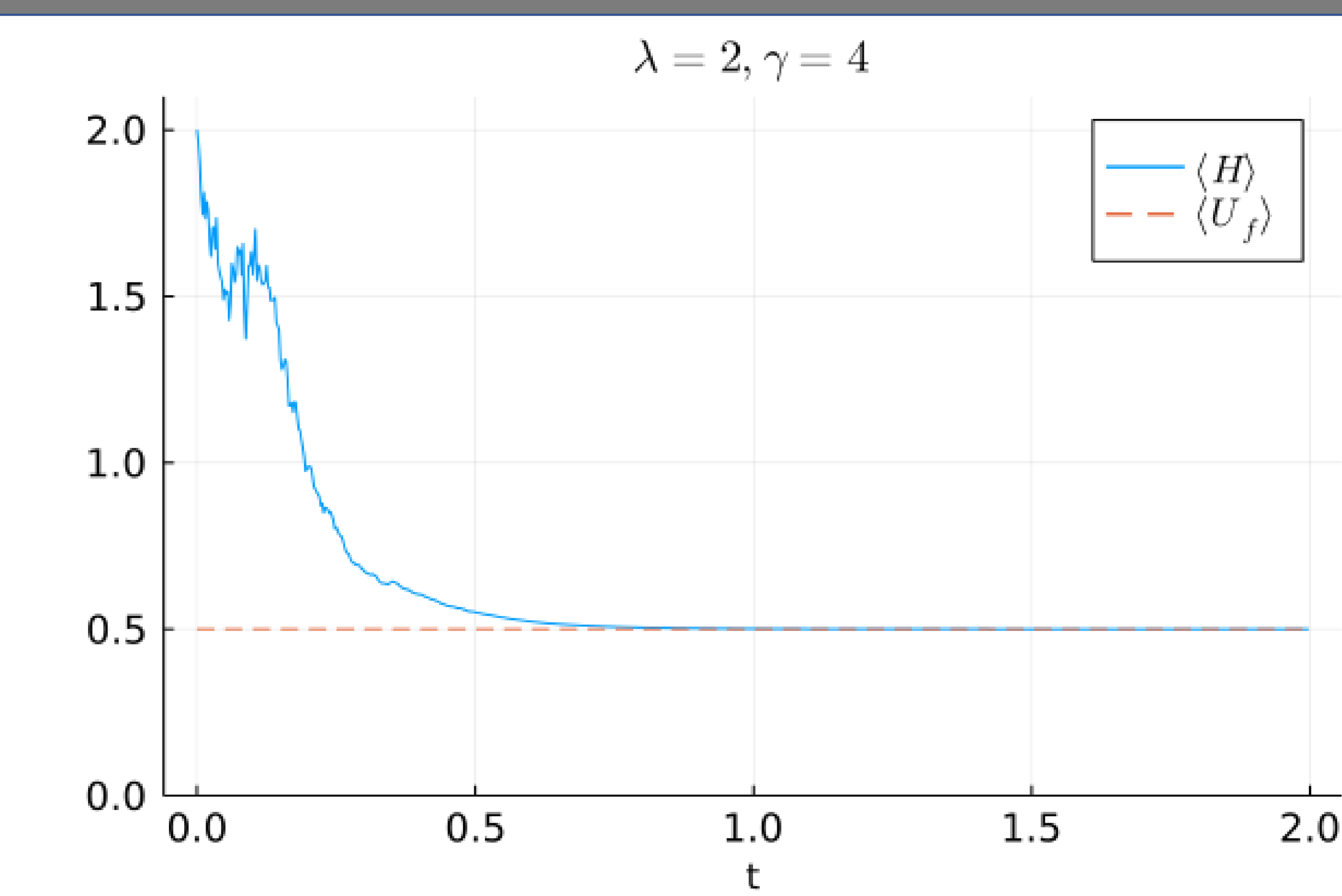

Fig 2. Single trajectory simulation for ground state cooling. We have stochastic evolution during some transient period, once that happens the energy evolves deterministically, and we achieve ground state cooling at the trajectory level.

Thermal coupling

- Now consider

$$\mathcal{L}(D_x,D_p)\rho = -i[H(D_x,D_p),\rho] + \Gamma(\bar{n}+1)\mathcal{D}[a]\rho + \Gamma\bar{n}\mathcal{D}[a^\dagger]\rho$$

- Instantaneous D_x, D_p-dependent basis

$$a = \frac{1}{\sqrt{2}}(x - D_x + ip + iD_p) \qquad \bar{n} = \left(e^{\beta\omega}-1\right)^{-1}$$

- Work: $\langle\mathcal{W}\rangle = -dt\langle H\mathcal{F}\rangle$ in terms of the Fokker-Planck term in QFPME
- Net work in the steady state is equal to the classical heat

$$\langle\mathcal{W}_{net}\rangle = \langle\mathcal{W}\rangle - \langle\mathcal{Q}_q\rangle = \langle\mathcal{Q}_{cl}\rangle$$

- Extracted power from a single reservoir

$$P = \frac{2\gamma\Gamma}{2\gamma+\Gamma}\hbar\omega\left[\left(\bar{n}+\frac{1}{2}\right) - \frac{1}{2}\left(\frac{\lambda}{\gamma} + \frac{\gamma}{4\lambda}\right)\right]$$

$$P_{max} \to \Gamma\hbar\omega\bar{n}$$

Conclusion & References

Conclusion

- Some systems can be solved analytically using the QFPME when focusing on the average of expectation values
- Quantum harmonic oscillator with position and momentum measurement can yield ground state cooling
- Power can be extracted from a single reservoir using continuous measurement and feedback

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References

[1]Annby-Andersson et al, arXiv:2110.09159