# Uniformly accelerated Brownian particle in a bosonic field bath: temperature-dependent dissipation and frequency shift

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## Introduction

A fundamental property of quantum field theory in Minkowski spacetime is that all inertial observers agree on the notion of the vacuum and the number of particles in a given field state. This is not the case for non-inertial observers, because different non-inertial observers define particles with respect to different field modes.

The most important example is the Unruh effect [2], which asserts that observers moving with a constant acceleration of magnitude $\alpha$ in Minkowski spacetime perceive the Minkowski vacuum as a thermal state at a temperature proportional to their acceleration, known as the Unruh temperature

$$T_U = \frac{\alpha}{2\pi}.$$

To put it another way, a uniformly accelerated observer through the Minkowski vacuum responds in exactly the same way as an inertial one immersed in a thermal reservoir at the Unruh temperature. However, this equivalence between the two scenarios has been reported to cease to hold when the dimensions of the background spacetime are odd [3].

We ask: *What this non-equivalence in odd spacetime dimensions implies for the dynamics of an accelerated Brownian particle?*

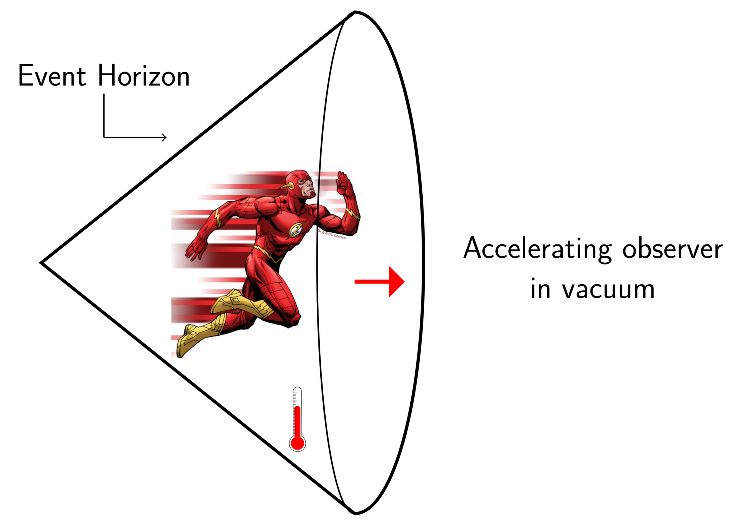



**Figure 1:** For a uniformly accelerated observer the Minkowski vacuum appears as a heat bath at the Unruh temperature $T_U = \alpha/2\pi$.

## The quantum Langevin equation

We consider a harmonic oscillator with unit mass and bare frequency $\Omega$, whose position operator $\hat{x}$ is linearly coupled to a massless quantum scalar field

$$\hat{\Phi}(t,\mathbf{x}) = \int \frac{d^n\boldsymbol{k}}{\sqrt{(2\pi)^n 2|\boldsymbol{k}|}} \left( \hat{a}_{\boldsymbol{k}} e^{-i(|\boldsymbol{k}|t - \boldsymbol{k}\cdot\mathbf{x})} + \text{H.c.} \right),$$

through the Hamiltonian [4]

$$H_{\text{int}} = \lambda \hat{x} \otimes \hat{\Phi}(\mathbf{x}),$$

where $\lambda$ is the coupling constant and $\hat{\Phi}(\mathbf{x})$ is the pullback of the field to the oscillator's position $\mathbf{x}$.

Working in the Heisenberg picture, the time evolution of the oscillator's position operator is given by the quantum Langevin equation

$$\ddot{\hat{x}}(\tau) + \Omega^2 \hat{x}(\tau) + 2\int_0^\tau ds\, \chi(\tau - s)\hat{x}(s) = \hat{\varphi}(\tau),$$

where $\hat{\varphi}(\tau) := \lambda\hat{\Phi}(\mathrm{x}(\tau))$ plays the role of a fluctuating force that obeys Gaussian statistics. The oscillator's worldline $\mathrm{x}(\tau) = (t(\tau), \mathbf{x}(\tau))$ is parametrized by its proper time $\tau$. A uniformly accelerated particle follows the hyperbolic trajectory

$$\mathrm{x}(\tau) = (\alpha^{-1}\sinh(\alpha\tau), \alpha^{-1}\cosh(\alpha\tau), x_\perp),$$

where $x_\perp$ denotes the spatial coordinate transverse to the direction of the acceleration.

We introduce the Wightman two-point correlation function of the field

$$\mathcal{W}(\tau,\tau') = \langle \hat{\varphi}(\tau)\hat{\varphi}(\tau')\rangle \equiv \nu(\tau,\tau') + i\chi(\tau,\tau')$$

evaluated along the oscillator's trajectory, where

$$\nu(\tau,\tau') := \frac{1}{2}\langle\{\hat{\varphi}(\tau),\hat{\varphi}(\tau')\}\rangle$$

is the *noise kernel*, and

$$\chi(\tau,\tau') := -\frac{i}{2}\langle[\hat{\varphi}(\tau),\hat{\varphi}(\tau')]\rangle$$

is the *dissipation kernel.*

## Correlation functions

| Dimensions | Brownian oscillator | Wightman function |
|---|---|---|
| (2+1) | inertial in thermal field bath | $\mathcal{W}_{\text{th}}(t) = \frac{\lambda^2}{4\pi}\int_0^\infty d\omega \left( \coth\left(\frac{\omega}{2T}\right)\cos(\omega t) - i\sin(\omega t)\right)$ |
| | accelerated in vacuum | $\mathcal{W}_{\text{acc}}(t) = \frac{\lambda^2}{4\pi}\int_0^\infty d\omega \left( \cos(\omega t) - i\tanh\left(\frac{\omega}{2T_U}\right)\sin(\omega t)\right)$ |
| (3+1) | inertial in thermal field bath | $\mathcal{W}_{\text{th}}(t) = \frac{\lambda^2}{4\pi^2}\int_0^\infty d\omega\, \omega \left( \coth\left(\frac{\omega}{2T}\right)\cos(\omega t) - i\sin(\omega t)\right)$ |
| | accelerated in vacuum | $\mathcal{W}_{\text{th}}(t) = \frac{\lambda^2}{4\pi^2}\int_0^\infty d\omega\, \omega \left( \coth\left(\frac{\omega}{2T_U}\right)\cos(\omega t) - i\sin(\omega t)\right)$ |

## Dissipation and frequency shift

We find that both the Brownian particle's dissipation rate

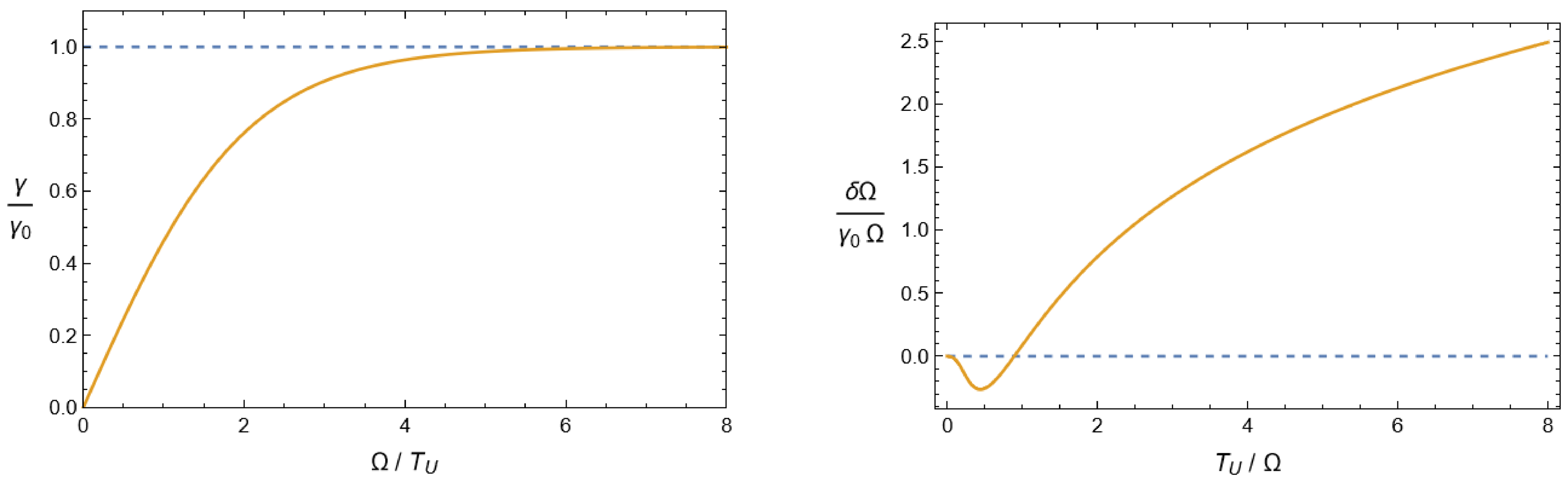
$$\gamma = \gamma_0 \tanh\left(\frac{\Omega}{2T_U}\right),$$

where $\gamma_0 = \frac{\lambda^2}{8\Omega}$ denotes the damping constant obtained in the case of inertial motion in field bath at a thermal state, and the shift of its frequency caused by the coupling to the field

$$\delta\Omega = \frac{4\gamma_0\Omega}{\pi}\left[\ln\left(\frac{2\pi T_U}{\Omega}\right) + \text{Re}\psi\left(\frac{i\Omega}{2\pi T_U} + \frac{1}{2}\right)\right]$$

depend on the acceleration temperature.

This is in contrast to the (3+1)-dimensional case, where dissipation and frequency shift do not exhibit temperature dependencies.




**(a)** Dissipation rate **(b)** Frequency shift

**Figure 2:** Dissipation rate and frequency shift of a uniformly accelerated oscillator detector (solid line) in the (2+1)-dimensional space compared to the ones found in the case of an inertial detector immersed in a heat bath at the Unruh temperature $T_U$ (dashed line).

## Late-time covariances

The fluctuation-dissipation theorem for the Brownian particle-field system

$$\widetilde{\nu}(\omega) = \coth\left(\frac{\omega}{2T_U}\right)\text{Im}\widetilde{\chi}(\omega),$$

holds for every spacetime dimension. At lates times and in the ultra-weak coupling limit the oscillator's covariance matrix takes the form

$$\langle\hat{x}^2(\infty)\rangle = \frac{1}{2\Omega}\coth\left(\frac{\Omega}{2T_U}\right),$$

$$\langle\hat{p}^2(\infty)\rangle = \frac{\Omega}{2}\coth\left(\frac{\Omega}{2T_U}\right),$$

which describe a thermal state at the Unruh temperature $T_U$.

Thus, a uniformly accelerated detector in Minkowski vacuum and an inertial one immersed in a thermal field bath at the Unruh temperature behave in the same way no matter what the dimensions of the background spacetime are only in terms of their late time behavior.

## References

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[4] B. L. Hu, S.-Y. Lin, and J. Louko, " Relativistic quantum information in detectors-field interactions", Class. Quantum Grav. 29, 224005 (2012).