Violation of detailed balance in open quantum systems

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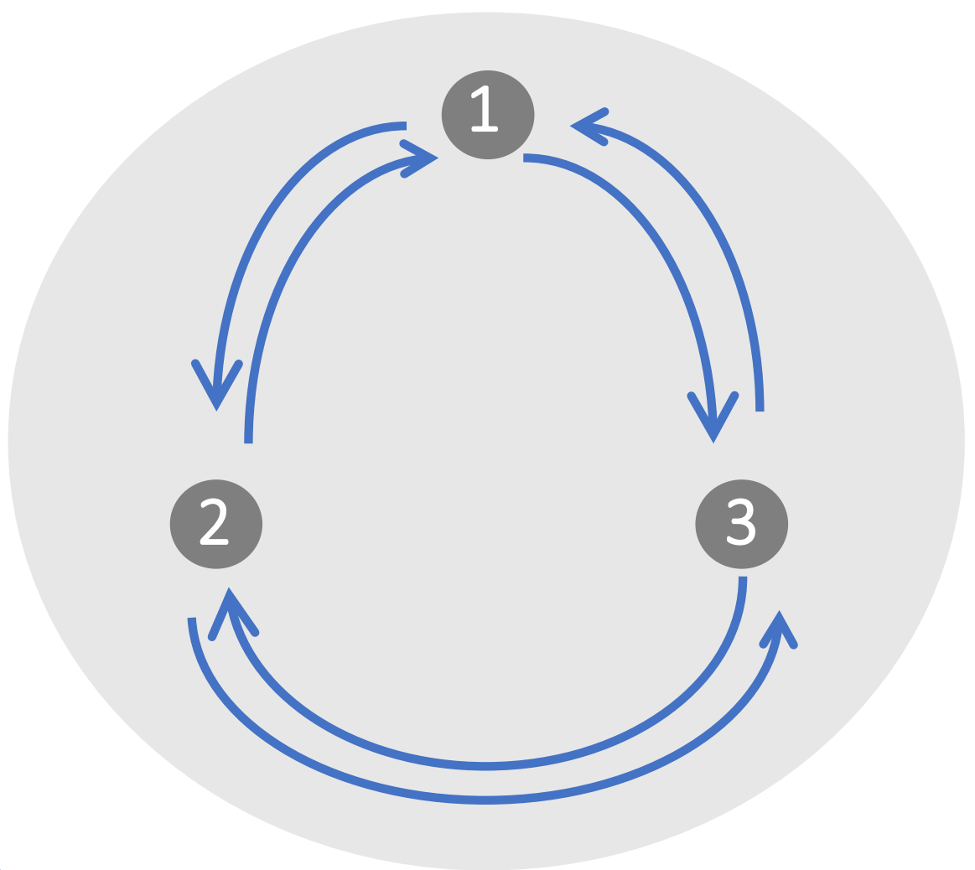
MOTIVATION

Detailed balance at thermal equilibrium: a common assumption



p_k state population

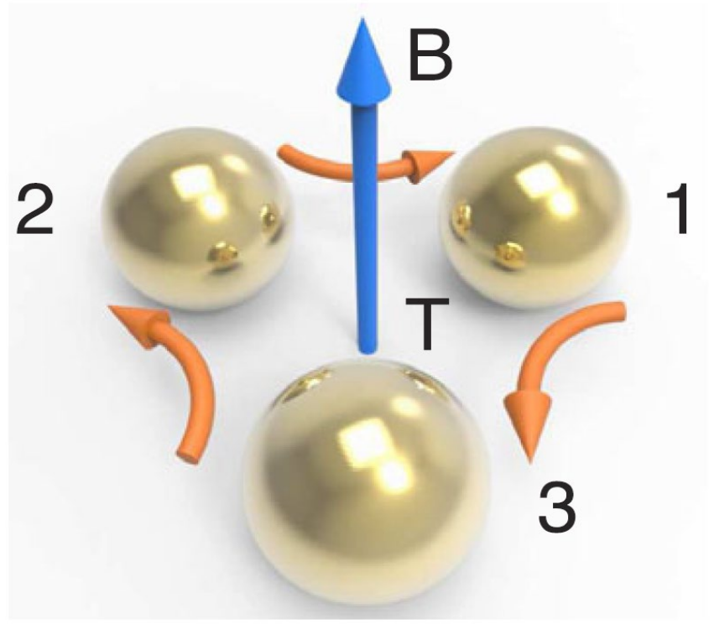
a_{jk} transition rate from k to j

Detailed balance:

$$a_{jk}p_k = a_{kj}p_j$$

What happens if it does not hold?

Heat currents at equilibrium[1]



Non-reciprocal systems break detail balance at thermal equilibrium

Open thermodynamic questions

Could non-reciprocal systems reach thermal equilibrium?
Yes: [2],[3],[4]; No:[5],[6],[7],[8]

Besides heat currents, forces are acting between the objects. What do we know regarding work extraction?

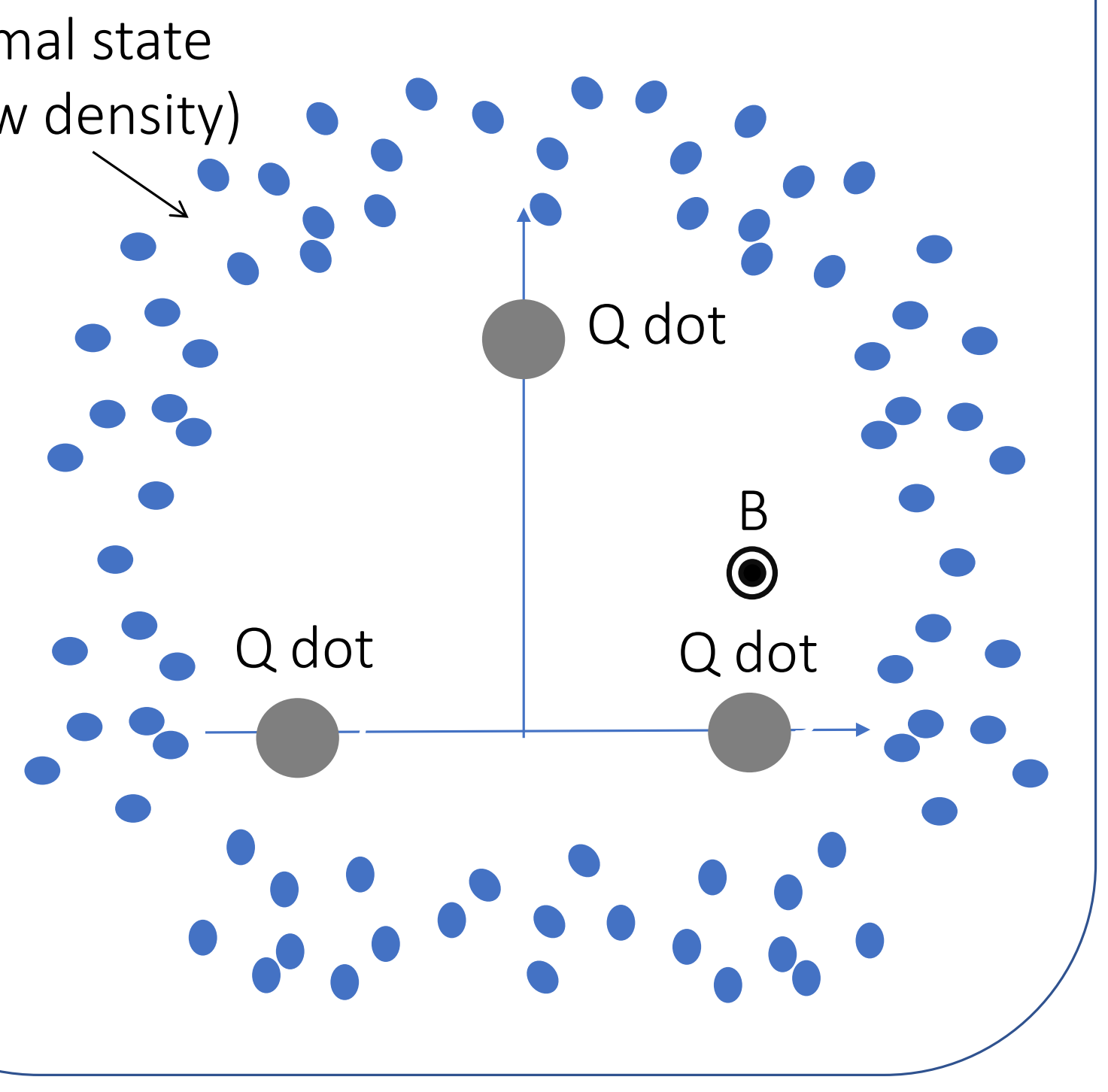
$$\sigma = \sum_{i>j}(P_i a_{ij} - P_j a_{ji})Log\left[\frac{P_i a_{ij}}{P_j a_{ji}}\right]$$

What is the entropy production of
a non-reciprocal system at thermal equilibrium?

LINDBLAD EQUATION, THERMODYNAMICS AND PHYSICAL MECHANISMS

Non-reciprocal toy model

2D free particle gas (thermal state and low density)



Lindblad equation for non-reciprocal systems [9]

$$\mathcal{L}\rho = \underset{\text{Bath Particle density}}{\pi\nu} \underbrace{\sum_{\omega=\epsilon_k-\epsilon_l}}_{\text{Sum over Bohr frequencies}} \overset{\text{Tracing the bath}}{\int d^2p\int d^2p'}\overbrace{Z^{-1}e^{-\beta E_p}}^{\text{Bath thermal state}}\overset{\text{Energy conservation}}{\delta(E_{p'} - E_p + \omega)} \times \left\{\left[T_\omega(p',p)\rho, T_\omega^\dagger\ (p',p)\right] + \left[\underset{\text{T-Matrix}}{T_\omega(p',p)}, \rho T_\omega^\dagger\ (p',p)\right]\right\}$$

$$T_\omega(p',p) = \sum_{\epsilon_k-\epsilon_l=\omega}\langle k,p'|T|p,l\rangle|k\rangle\langle l|$$

T-Matrix represents the scattering amplitude

Non-reciprocal systems reach thermal equilibrium!

$\mathcal{L}\rho_{ss}$=0 $\rho_{ss} = Z_s^{-1}e^{-\beta H_s}$

The steady-state of the Lindblad equation is a thermal state even for **broken** detailed balance!

The physics behind breaking detailed balance

$a_{kl}e^{-\beta\epsilon_l} = a_{lk}e^{-\beta\epsilon_k}I(k,l) = 1$ → Detailed balance

$$I(k,l) = \frac{\int d^2p\int d^2p' Z^{-1}e^{-\beta E_p}\delta(E_{p'} + \epsilon_k - E_p - \epsilon_l)|\langle k,p'|T|p,l\rangle|^2}{\int d^2p\int d^2p' Z^{-1}e^{-\beta E_p}\delta(E_{p'} + \epsilon_k - E_p - \epsilon_l)|\langle l,p|T|p',k\rangle|^2}$$

Each of the following conditions will imply detailed balance:

1) Time-reversal symmetry: $E_p = E_{-p}, |k\rangle$ symmetric and
$|\langle k,p'|T|p,l\rangle|^2$= $|\langle l,-p|T|-p',k\rangle|^2$

2) Hermiticity: $\langle k,p'|\ T|p,l\rangle$= $\langle l,p|T|p',k\rangle^*$

Beyond weak coupling: detailed balance at equilibrium breaks down

$$T_{\omega=\epsilon_i-\epsilon_j}(p',p) = \underbrace{\langle i,p'|H_{int}|p,j\rangle}_{\text{Weak coupling}} + O(H_{int}^2)$$

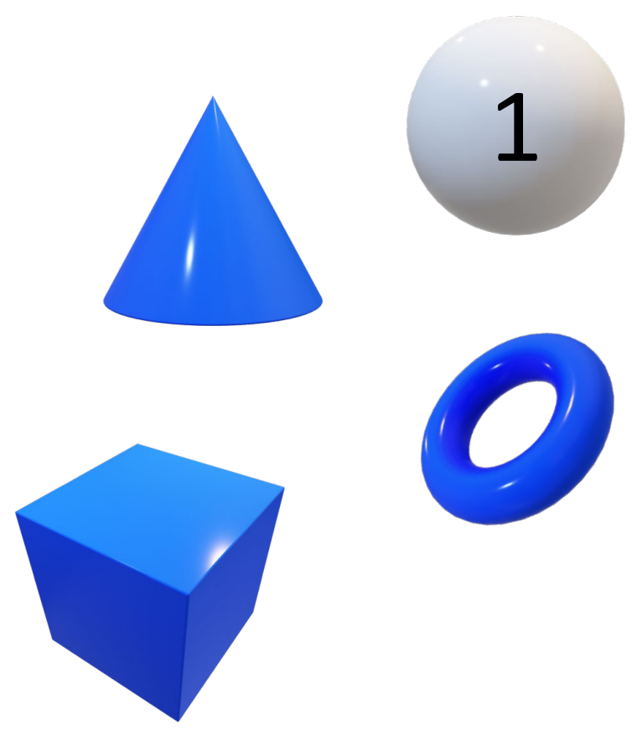
Microreversibility breaks at first order on H_{int} : $|\langle 0,p'|T|p,+\rangle|^2 \neq |\langle +,-p|T|-p',0\rangle|^2$

Hermiticity breaks at second order on H_{int} : $\langle 0,p'|\ T|p,+\rangle$= $\langle +,p|T|p',0\rangle^*$

$a_{0+}e^{-\beta E_+} \neq a_{+0}e^{-\beta E_0}$ This system breaks detailed balance!

CASIMIR FORCES AND WORK EXTRACTION

Equilibrium Casimir forces are conservative



If all the bodies and their surroundings are at the same temperatures, one can show that the force on object one (or any other) can be written as:

$$F^1 = -\nabla_{O_1}f$$

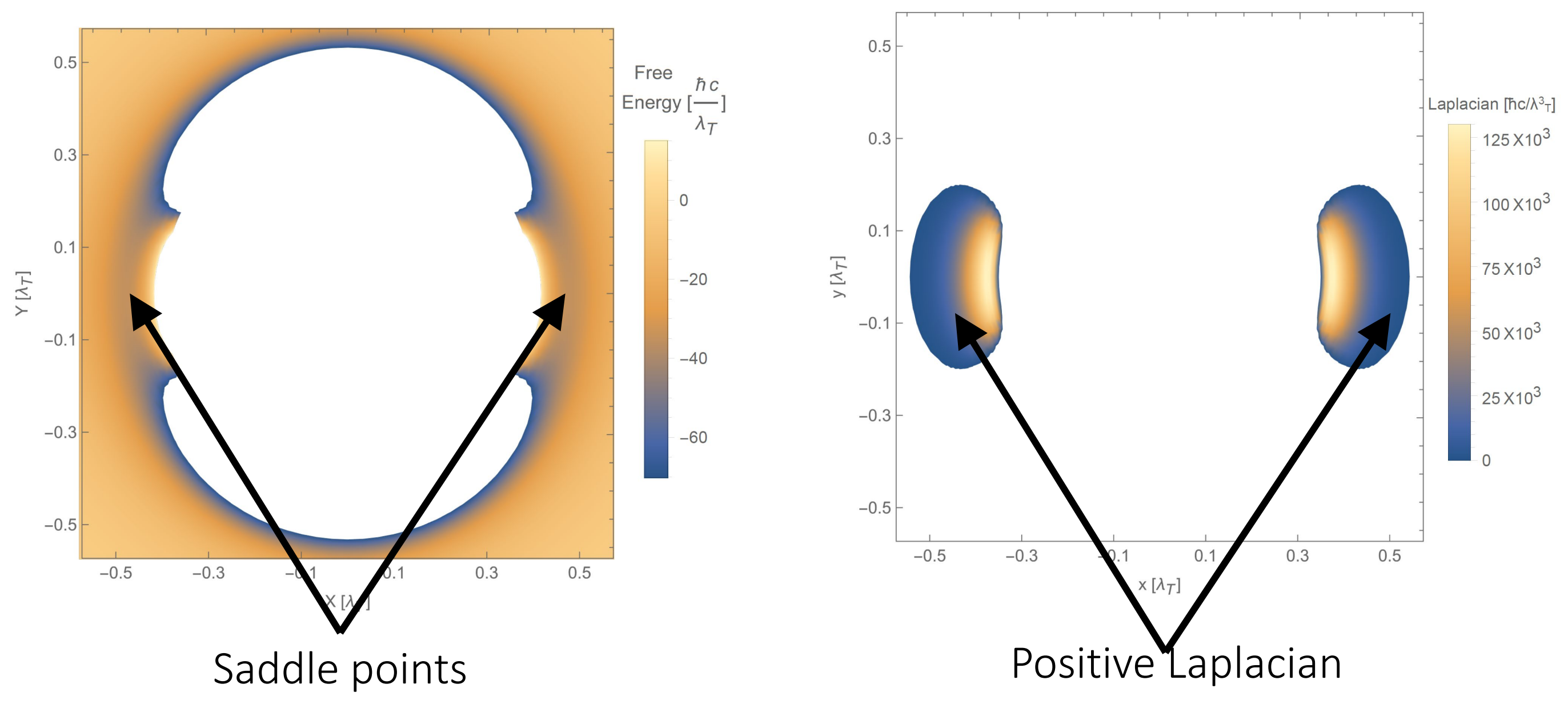
f plays the role of a potential or free energy

The force is conservative forbidding cyclic work extraction [10]

Cyclic work extraction possible outside equilibrium [11]

Do non-reciprocal systems fulfill Earnshaw's theorem?

For reciprocal systems, the non-positivity of the Laplacian is used to prove Earnshaw's theorem.
This can no longer be used for non-reciprocal systems



Saddle points

Positive Laplacian

Conclusions

Non-reciprocal systems break detailed balance at thermal equilibrium. These allow them to sustain steady heat currents even at equilibrium. We developed a Lindblad equation appropriate for studying these systems and clarifying several of their thermodynamic properties. While non-reciprocal systems can sustain persistent Heat currents, they can reach thermal equilibrium and the forces they produce at thermal equilibrium Are conservative. In contrast, for non-equilibrium situations, non-reciprocal systems can be used to build a Casimir force heat engine.

We are looking for students/postdocs

[1] L, Zhu, et. al. *PRL* 117 (2016): 134303; [2] V. Asadchy, et. al. Proc. IEEE 108 (2020): 1684; [3] S.A. Mann, et. al. Optica 7 (2020): 1102; [4] C. Khandekar, et. al. NJP 22 (2020): 123005;[5] S.-H. Chong, et. al. PRE E 81 (2010): 041130; [6] P. Reimann, et. al. Appl. Phys. A 75 (2002): 169; [7] P. Hänggi, et. al. Phys. Rep. 88 (1982) : 207; [8] S.A. Loos, et. al. NJP 22 (2020): 123051; [9] D.G.-K, et al. (2022) *In preparation;* [10] D.G.-K, et al. (2022) arXiv:2205.13470 [quant-ph] (2022)*;* [11] D.G.-K, et al. *PRL* 126 (2021): 170401.