Operational models of temperature superpositions†

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Temperature Superpositions?

- How can quantum systems thermalise with baths whose temperature varies locally?
- Can temperature display quantum features?
- Can probes be placed in a `superposition of temperatures?

→ Here: Explore two possible operational models.

Thermal Channels/Thermalisation

- Probe interacts with bath via unitary evolution
  → Thermalises to temp. of bath; $\rho_{in} = \rho_S \otimes \rho_B$

$$\rho_{in} \,\text{—}\boxed{\text{U}}\text{—}\, \rho_{out}$$

- Effective map on probe (LHS) as Kraus decomp. (RHS); $M_{kl} = \langle k|U_{SB}|l\rangle$:

$$\mathcal{E}(\rho_S) = \sum_{k,l} c_l^B M_{kl} \rho_S M_{kl}^\dagger$$

1: Two Baths (Superposition of Channels)

- Quantum-controlled interactions with baths, (like interferometer)
- Initial state

$$\rho = \underset{\text{Probe}}{\rho_S} \otimes \overbrace{(\rho_{B_0} \otimes \rho_{B_1})}^{\text{Baths}} \otimes \overset{\text{Control}}{\rho_C}$$

- Final state (after measuring control):

$$\boxed{\rho_S^{\beta_i} = \sum_k c_k^{\beta_i} |k\rangle_S \langle k|_S ; \beta_i \equiv \frac{1}{T_i}}$$

$$\rho_S^{(\phi)} = \frac{1}{4}\left(\rho_S^{\beta_0} + \rho_S^{\beta_1} + e^{i\phi} \rho_S^{\beta_0} u^0 \rho_S u^{1\dagger} \rho_S^{\beta_0} + \text{H.c.}\right)$$

- Not thermal
- Dependent on both bath states + initial probe state
- and on local bath unitaries u^i

2: One Bath (Superposition of Purifications)

- Superposition of purifications: $|\tilde{\psi}\rangle = \sum_x |\theta^\beta(x)\rangle \, \overset{\overset{\text{Control}}{\uparrow}}{|x\rangle_C}$
  - Purifications have form:

$$|\theta^\beta(x)\rangle = \sum_b e^{-i\phi_x} \sqrt{c_b^{\beta_x}} \, \underset{\swarrow \text{Bath}}{|b\rangle_B} \, |a(b,x)\rangle_A \leftarrow \text{Ancilla}$$

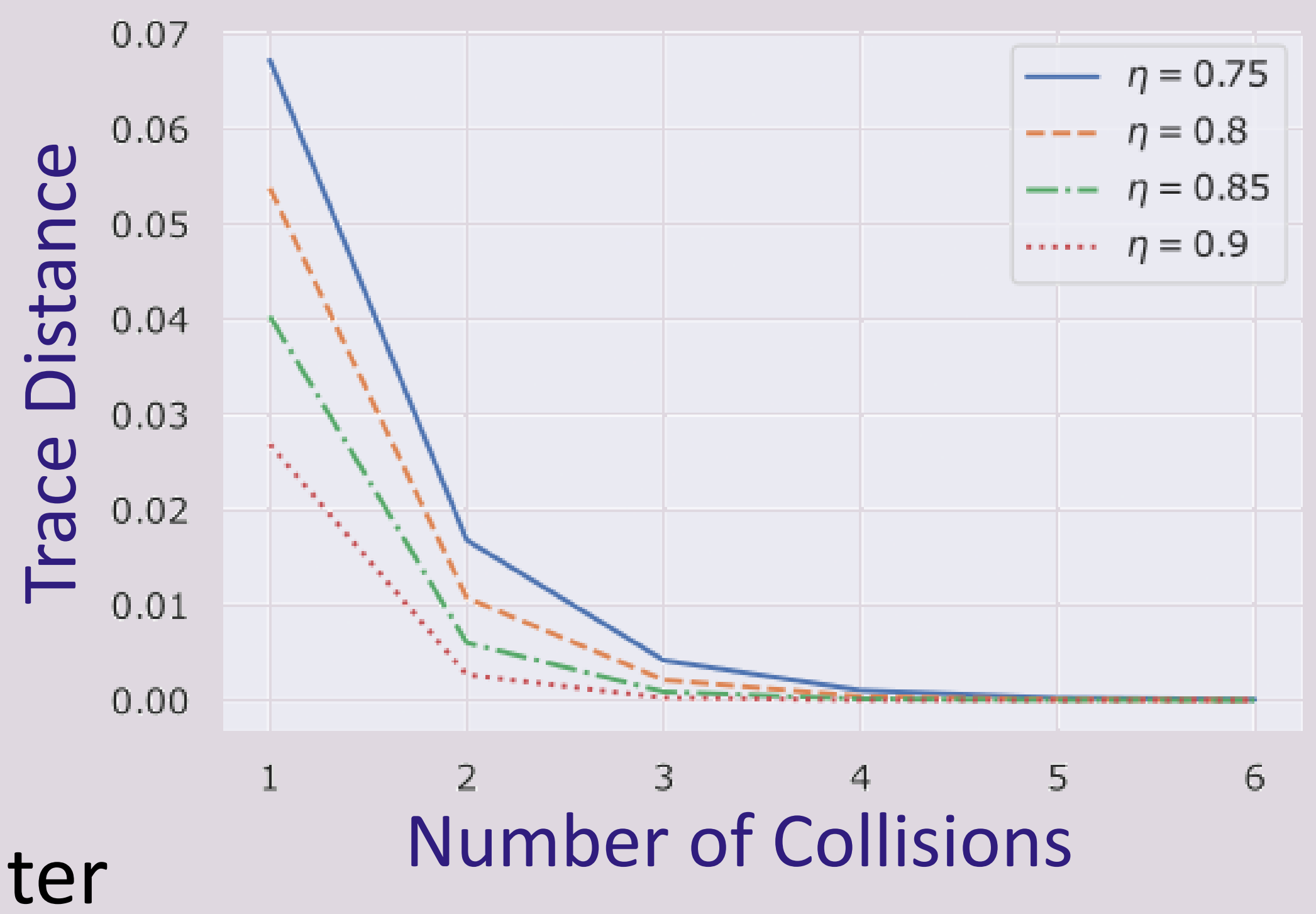
  - Control state determines bath temperature
- Final state:

$$\tilde{\rho}_S^\phi = \frac{1}{4}(\rho_S^{\beta_0} + \rho_S^{\beta_1} + e^{-i\tilde{\phi}} \sqrt{c_b^{\beta_0} c_{b'}^{\beta_1}} \mathrm{Tr}_S\{u_{SA}^0 \rho_S u_{SA}^{1\dagger}\} + \text{H.c.})$$

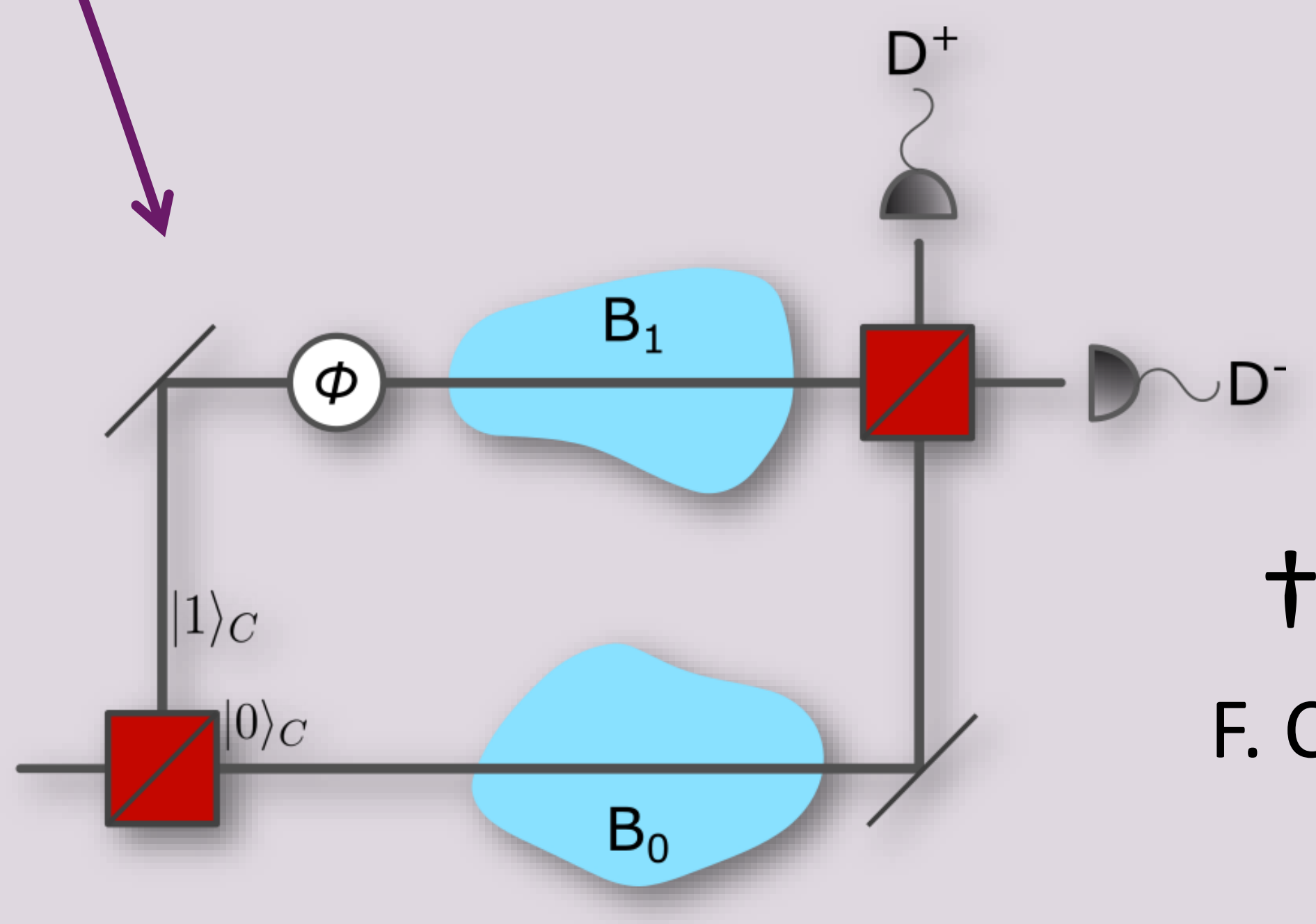
  - Can be thermal (if β_0=β_1)
  - Still dependent on local unitaries

Partial and Pre-Thermalisation

- Gradual thermalisation → Successively apply U_{SB}
- Interactions last finite time, with interaction parameter $\eta < 1$

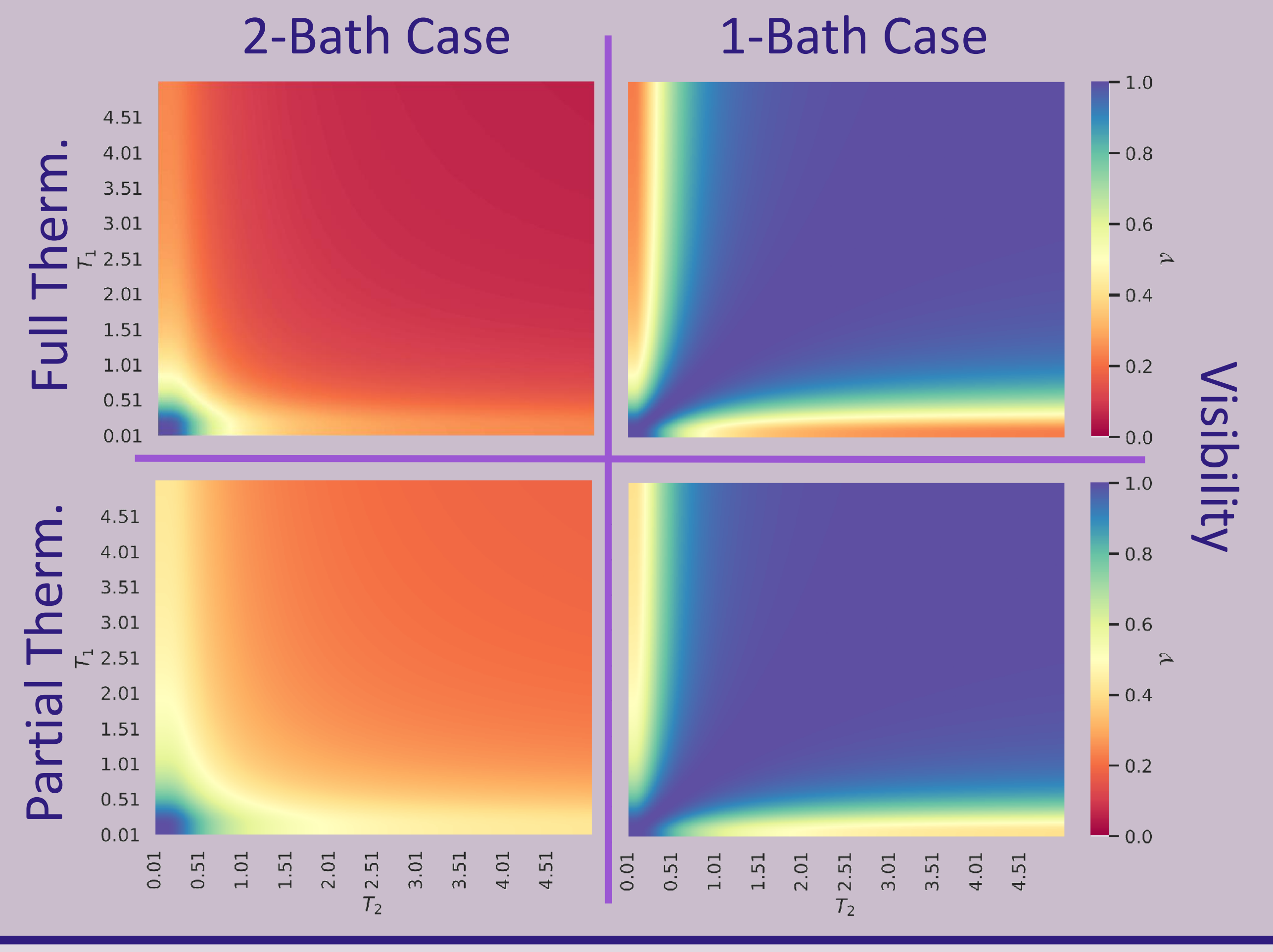


- Changes in η alter no. of collisions to full thermalisation



Comparing Cases through Visibility

- Visibility as indicator of ‘temperature coherence’
- Two-bath case $\quad \mathcal{V} = \left|\mathrm{Tr}\left\{\rho_S^{\beta_0} \rho_S^{\beta_1} u^1 \rho_S u^{0\dagger}\right\}\right|$
  - No thermalisation
  - Low visibility, except at low temps
- One-bath case $\quad \tilde{\mathcal{V}} = \sum_b \sqrt{c_b^{\beta_0} c_b^{\beta_1}} \, V_{bb}^{01} |\mathrm{Tr}_S\{u^0 \rho_S u^{1\dagger}\}|$
  - Thermalisation possible
  - Max. visibility possible, except for one temp. low



Summary and Conclusions

- Two cases of a ‘Superposition of Temperatures’
  - Two separated baths: no thermalisation
  - One bath in superposition: thermalisation can be reached
  - Both depend on the form of the interaction
- Implications:
  - Greater understanding of bath dynamics?
  - Sensitivity to local unitaries
- Future directions
  - Unruh-DeWitt detectors
  - Tolman-Ehrenfest effect

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† C.E. Wood, H. Verma, F. Costa & M. Zych (2021), arXiv:2112.07860

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