Sphalerons from the compact 341 model and baryogenesis

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Abstract

The sphaleron energy and rate investigated within the compact 341 model. It is shown, that the constraint leading to the first order phase transition is satisfied for the three spontanous symmetry breaking vaccums of the model.

I- Introduction

The Standard Model (SM) of particles physics cannot explain the observed baryonic asymmetry of the universe and the electroweak phase transition (EWPT) is of first order only if the mass of the Higgs boson is less than 70 GeV. This is in contradiction with the current experimental value which is around 125 GeV. In the compact 341 model, the first order (EWPT) is satisfied. An accurate calculation of the sphaleron energy is important for assessing the viability of the (EWPT) because the baryon number condition typically written $2E/\lambda \geq 1$ is directly proportional to the sphaleron energy.

II- Particles content of the compact 341 model

We have three generations of fermions represented by the quartets [1]

$$L_{aL} = \begin{pmatrix} \upsilon_a \\ l_a \\ \upsilon_a^c \\ l_a^c \end{pmatrix}_L \text{ with } a = e,\mu,\tau,\; Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ U_1 \\ J_1 \end{pmatrix}_L,\; Q_{iL} = \begin{pmatrix} d_i \\ u_i \\ D_i \\ J_i \end{pmatrix}_L \text{ and } i = 2,3 \tag{1}$$

The scalar potential is given by:

$$\begin{aligned} V(\eta,\rho,\chi) = {} & \mu_\eta^2\eta^+\eta + \mu_\rho^2\rho^+\rho + \mu_\chi^2\chi^+\chi + \lambda_1(\eta^+\eta)^2 + \lambda_2(\rho^+\rho)^2 + \lambda_3(\chi^+\chi)^2 \\ & + \lambda_4(\eta^+\eta)(\rho^+\rho) + \lambda_5(\eta^+\eta)(\chi^+\chi) + \lambda_6(\rho^+\rho)(\chi^+\chi) + \lambda_7(\rho^+\eta)(\eta^+\rho) \\ & + \lambda_8(\chi^+\eta)(\eta^+\chi) + \lambda_9(\rho^+\chi)(\chi^+\rho) + h.c \end{aligned} \tag{2}$$

where $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7,\ \lambda_8,\ \lambda_9$ are the dimensionless coupling constants and $\mu^2_{\eta,\rho,\chi}$ are the mass dimension parameters. $\eta,\ \rho$ and χ are the three Higgs scalar quartets:

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_1^- \\ \eta_2^0 \\ \eta_2^+ \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(R_{\eta_1} + iI_{\eta_1}) \\ \eta_1^- \\ \frac{1}{\sqrt{2}}(\upsilon_\eta + R_{\eta_2} + iI_{\eta_2}) \\ \eta_2^+ \end{pmatrix},\ \rho = \begin{pmatrix} \rho_1^+ \\ \rho^0 \\ \rho_2^+ \\ \rho^{++} \end{pmatrix} = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(\upsilon_\rho + R_\rho + iI_\rho) \\ \rho_2^+ \\ \rho^{++} \end{pmatrix},$$
$$\chi = \begin{pmatrix} \chi_1^- \\ \chi^{--} \\ \chi_2^- \\ \chi^0 \end{pmatrix} = \begin{pmatrix} \chi_1^- \\ \chi^{--} \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\upsilon_\chi + R_\chi + iI_\chi) \end{pmatrix}. \tag{3}$$

We have three steps of symmetry breaking, when χ^0develops a vacuum expectation value (VeV) υ_χ, ρ^0 develops a (VeV) υ_η and ρ^0 develops a (VeV) υ_ρ:

$$SU(4)_L \otimes U(1)_X \underset{\upsilon_\chi}{\to} SU(3)_L \otimes U(1)_X \underset{\upsilon_\eta}{\to} SU(2)_L \otimes U(1)_Y \underset{\upsilon_\rho}{\to} U(1)_{QED} \tag{4}$$

where the VEVs satisfy the constraints $\upsilon_\chi \approx \upsilon_\eta \gg \upsilon_\rho$. This model contains three neutral Higgses H_1^0, H_2^0, H_3^0 , two simply charged Higgses $h_1^\mp$, $h_2^\mp$ and a doubly charged Higgs h^{++} with the following masses:

$$\begin{aligned} M^2_{H_1^0} &= \left(\lambda_2 + \frac{[\lambda_3\lambda_4^2 + \lambda_6(\lambda_1\lambda_6 - \lambda_4\lambda_5)]}{\lambda_5^2 - 4\lambda_1\lambda_3}\right)\upsilon_\rho^2,\ M^2_{H_2^0} = \frac{1}{2}\left(\lambda_1 + \lambda_3 - \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_5^2}\right)\upsilon_\eta^2, \\ M^2_{H_3^0} &= \frac{1}{2}\left(\lambda_1 + \lambda_3 + \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_5^2}\right)\upsilon_\chi^2,\ M^2_{h_1^\mp} = \frac{1}{2}\lambda_7(\upsilon_\eta^2 + \upsilon_\rho^2),\ M^2_{h_2^\mp} = \frac{1}{2}\lambda_8(\upsilon_\eta^2 + \upsilon_\chi^2),\ M^2_{h^{\pm\pm}} = \frac{1}{2}\lambda_9(\upsilon_\rho^2 + \upsilon_\chi^2). \end{aligned} \tag{5}$$

The lagrangien density which gives the masses of gauge bosons is given by:

$$\mathcal{L}^B = (D_\mu\chi)^+(D^\mu\chi) + (D_\mu\eta)^+(D^\mu\eta) + (D_\mu\rho)^+(D^\mu\rho), \tag{6}$$

and $D^\mu = \partial^\mu - \frac{ig_L}{2}\lambda_\alpha A^\mu_\alpha - ig_x XB^\mu = \partial^\mu - iP^\mu$, $\alpha = 1,\ldots 15$ with λ_α the Gellmann matrices, X is the quantum number of the group $U(1)_X$, the gauge bosons masses:

$$\begin{aligned} & M^2_{W^\pm} = g_L^2\upsilon_\rho^2/4,\ M^2_{K_1^\mp} = g_L^2(\upsilon_\rho^2 + \upsilon_\eta^2)/4,\ M^2_{K,K'} = g_L^2\upsilon_\eta^2/4,\ M^2_{Y^\pm} = g_L^2(\upsilon_\rho^2 + \upsilon_\chi^2)/4,\ M^2_{V^{\pm\pm}} = g_L^2(\upsilon_\eta^2 + \upsilon_\chi^2)/4,\ M^2_{X^\pm} = g_L^2\upsilon_\chi^2/4 M_Z^2 = g_L^2\upsilon_\rho^2/4\cos^2\theta_W, \\ & M^2_{Z'} = g_L^2(\cos^2\theta_W\upsilon_\eta^2)/(3 - 4\sin^2\theta_W),\ M^2_{Z''} = g_L^2\upsilon_\eta^2[(1 - 4\sin^2\theta_W) + (3 - 4\sin^2\theta_W)]/8(3 - 4\sin^2\theta_W)(1 - 4\sin^2\theta_W). \end{aligned} \tag{7}$$

III- The Electroweak Phase Transition

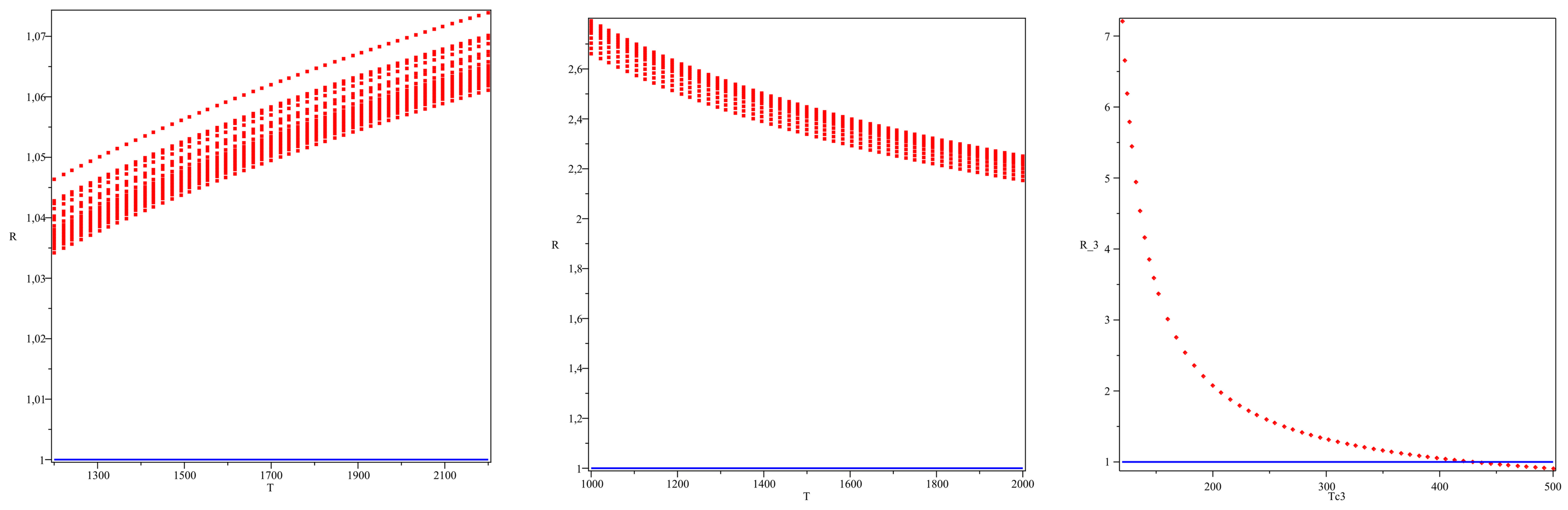
The one-loop effective potential at finite temperature in the $\overline{DR}$ renormalization scheme $\overline{DR}$ has the form [2]

$$\begin{aligned} V_{\text{eff}}(\phi,T) = {} & \frac{1}{2}(\mu_\chi^2\upsilon_\chi^2 + \mu_\eta^2\upsilon_\eta^2 + \mu_\rho^2\upsilon_\rho^2) + \frac{1}{4}(\lambda_1\upsilon_\eta^4 + \lambda_2\upsilon_\rho^4 + \lambda_3\upsilon_\chi^4 + \lambda_4\upsilon_\eta^2\upsilon_\rho^2 + \lambda_5\upsilon_\eta^2\upsilon_\chi^2 + \lambda_6\upsilon_\chi^2\upsilon_\rho^2) \\ & + \sum_{i=\text{bosons, fermions}} \tilde{n}_i \frac{m_i^4}{64\pi^2}\left[\ln\frac{m_i^2}{\mu^2} - \frac{3}{2}\right] + \frac{T^4}{2\pi^2}\sum_{i=\text{bosons, fermions}} \tilde{n}_i J_{B,F}\left(\frac{m_i}{T^2}\right), \end{aligned} \tag{8}$$

where $J_{B,F}$ represent the bosonic and fermionic thermal functions, m_i are the field-dependant masses, μ is the renormalization scale and $\tilde{n}_i$ are the field multiplicities $\tilde{n}_h = 1$, $\tilde{n}_{\text{charged}} = 2$, $\tilde{n}_{\text{quark}} = -12$, $\tilde{n}_{Z,Z',Z'',K,K'} = 3$, $\tilde{n}_{W,K_1,Y,X,V} = 6$.

In this model we have three VeVs take the values $\upsilon_\rho = 246$ GeV, $\upsilon_\chi \sim \upsilon_\eta \sim 2 - 5$ TeV in order to avoid the Landau pole and the masses of exotique quarks are in the range of $600 - 700$ GeV which are comptible with the LHC results and we have three steps of symmetry breaking in other word three first order phase transitions. The first two transitions in the TeV scale and only concern exotic quarks, heavy bosons without the involvement of SM particles, in these two steps particles acquires its masses by interacting respectively with the scalars fields of the first massive neutral Higgs H_3^0 and the second H_2^0. The third steps of (EWPT) in the GeV scale and only concern SM particles by interacting with the scalar field of SM Higgs H_1^0.

So, we have three conditions of (EWPT) $R = \upsilon_c/T_c = 2E/\lambda$ and $R \geq 1$, the following figures represents the ratio R in terms of T_c for the three steps and as it is shown this ratio fulfills the necessary condition for a strond first order (EWPT).



The ratio R in terms of Tc for the three steps of the phase transition

IV- Sphaleron rate in the compact 341 Model

Sphalerons are one of the most important ingredients in the study of electroweak baryogenesis (EWBG) because the sphaleron rate controls the rate of the baryon number density in the early universe, the sphaleron phenomenon occurs if one has the transition from the zero VeV crossing the barrier to a non-zero VeV without tunneling (classically).

The sphaleron rate Γ by unit time is related to the sphaleron energy ε via the relation [3]

$$\frac{\Gamma}{V} = \alpha^4 T^4 \exp(-\varepsilon/T) \tag{9}$$

with T is the temperature, $\alpha = 1/30$ is a constant and V is the volume of the EWPT region, $V = \frac{4\pi r^3}{3} \sim \frac{1}{T^3}$ and ε is the sphaleron energy that take the following form:

$$\varepsilon = 4\pi\int d^3x\left[\frac{1}{2}(\nabla\upsilon_\chi)^2 + \frac{1}{2}(\nabla\upsilon_\eta)^2 + \frac{1}{2}(\nabla\upsilon_\rho)^2 + V_{\text{eff}}(\chi,\eta,\rho,T)\right] \tag{10}$$

with $V_{\text{eff}}(\chi,\eta,\rho,T)$ is the effective potential at finite temperature for the three steps of phase transitions

$$\begin{aligned} V_{\text{eff}}(\upsilon_\chi,T) &= D(T^2 - T_0^2)\upsilon_\chi^2 - ET\upsilon_\chi^3 + \frac{\lambda(T)}{4}\upsilon_\chi^4 \\ V_{\text{eff}}(\upsilon_\eta,T) &= D'(T^2 - T_0'^2)\upsilon_\eta^2 - E'T\upsilon_\eta^3 + \frac{\lambda'(T)}{4}\upsilon_\eta^4 \\ V_{\text{eff}}(\upsilon_\rho,T) &= D''(T^2 - T_0''^2)\upsilon_\rho^2 - E''T\upsilon_\rho^3 + \frac{\lambda''(T)}{4}\upsilon_\rho^4 \end{aligned} \tag{11}$$

using the thin wall approximation

$$\frac{\partial V_{\text{eff}}(\upsilon_{\chi,\eta,\rho})}{\partial\upsilon_{\chi\eta,\rho}} = C_{\chi,\eta,\rho} = \text{const}, \tag{12}$$

m of V_{eff} in the bubble phase transition, the field equations of the VeVs $\upsilon_{\chi,\eta,\rho}$ read

$$\frac{d^2\upsilon_{\chi,\eta,\rho}}{dr^2} + \frac{2}{r}\frac{d\upsilon_{\chi,\eta,\rho}}{dr} = C_{\chi,\eta,\rho} \tag{13}$$

with the boundary conditions

$$\lim_{r\to\infty}\upsilon_{\chi,\eta,\rho}(r) = 0, \qquad \left.\frac{d\upsilon_{\chi,\eta,\rho}(r)}{dr}\right|_{r=0} = 0. \tag{14}$$

The solutions of Eqs. (13) and (14) are given by

$$\upsilon_{\chi,\eta,\rho} = \frac{C_{\chi,\eta,\rho}}{6}r^2 - \frac{A_{\chi,\eta,\rho}}{r} + B_{\chi,\eta,\rho}, \tag{15}$$

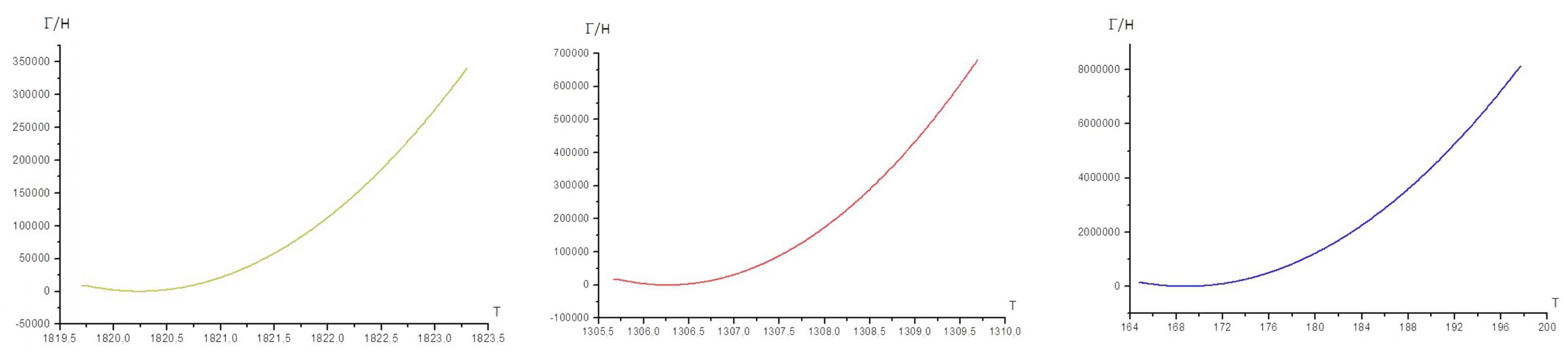
where $A_{\chi,\eta,\rho}$, $B_{\chi,\eta,\rho}$ are integration constants. To be more precise if the sphaleron has a thickness $\Delta l_{\chi,\eta,\rho}$ and radius $R_{\chi,\eta,\rho}$ the solution $\upsilon_{\chi,\eta,\rho}$ can be expressed like this

$$\upsilon_{\chi,\eta,\rho}(r) = \begin{cases} \upsilon_{\chi_c,\eta_c,\rho_c} & \text{when } r \leq R_{\chi,\eta,\rho}, \\ \frac{C_{\chi,\eta,\rho}}{6}r^2 - \frac{A_{\chi,\eta,\rho}}{r} + B_{\chi,\eta,\rho} & \text{when } R_{\chi,\eta,\rho} < r \leq R_{\chi,\eta,\rho} + \Delta l_{\chi,\eta,\rho}, \\ 0 & \text{when } R_{\chi,\eta,\rho} + \Delta l_{\chi,\eta,\rho} < r. \end{cases} \tag{16}$$

Here $\upsilon_{\chi_c,\eta_c,\rho_c}$ stands for the second minimum for the 3 steps of the phase transition. To continue the constant $C_{\chi,\eta,\rho}$ can be approximated as

$$C_{\chi,\eta,\rho} \sim \frac{\Delta V_{\text{eff}}(\upsilon_{\chi,\eta,\rho})}{\Delta\upsilon_{\chi,\eta,\rho}}, \tag{17}$$

where $\Delta V_{\text{eff}} = V_{\text{eff}_c}(\upsilon_{\chi_c,\eta_c,\rho_c})$ and $\Delta\upsilon_{\chi,\eta,\rho} = \upsilon_{\chi_c,\eta_c,\rho_c}$. In order to avoid the washout of the baryonic asymmetry after the phase transition one has to assume that the sphaleron rate Γ has to be equal to the Hubble parameter H at the critical temperature T_c. Of course Γ has to be lager than H at $T > T_c$ and smaller at $T < T_c$, for an illustration if $T = 1822, 1308, 180$ GeV, $\Gamma = 3.7084 \times 10^7, 2091.05642, 56.98872$ GeV and $\frac{\Gamma}{H} = 7.9379 \times 10^{18}, 8.6848 \times 10^{14}, 1.2498 \times 10^{15}$. Of course, at $T = T_c$ the sphaleron rate $\Gamma = H = 4.66178 \times 10^{-12}, 2.3913 \times 10^{-12}, 3.94764 \times 10^{-14}$GeV. When T becomes smaller than T_c ($T = 1811.5, 1300, 158$ GeV) Γ decreases rapidly and the ratio $\frac{\Gamma}{H}$ becomes less than 1 ($\frac{\Gamma}{H} = 3.19962 \times 10^{-210}, 3.2444 \times 10^{-19}, 1.8254 \times 10^{-109}$).



The ratio $\frac{\Gamma}{H}$ as a function of T for the three steps of the phase transition

V- Conclusion

We have investigated the possibility that the compact 341 model explain the (EWBG), we have calculated Γ and we have compared with H which describe the cosmological expansion rate at T, we have proved that Γ fulfills the condition of a strong first order (EWPT).

References

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