Catalysis in Action via Elementary Thermal Operations

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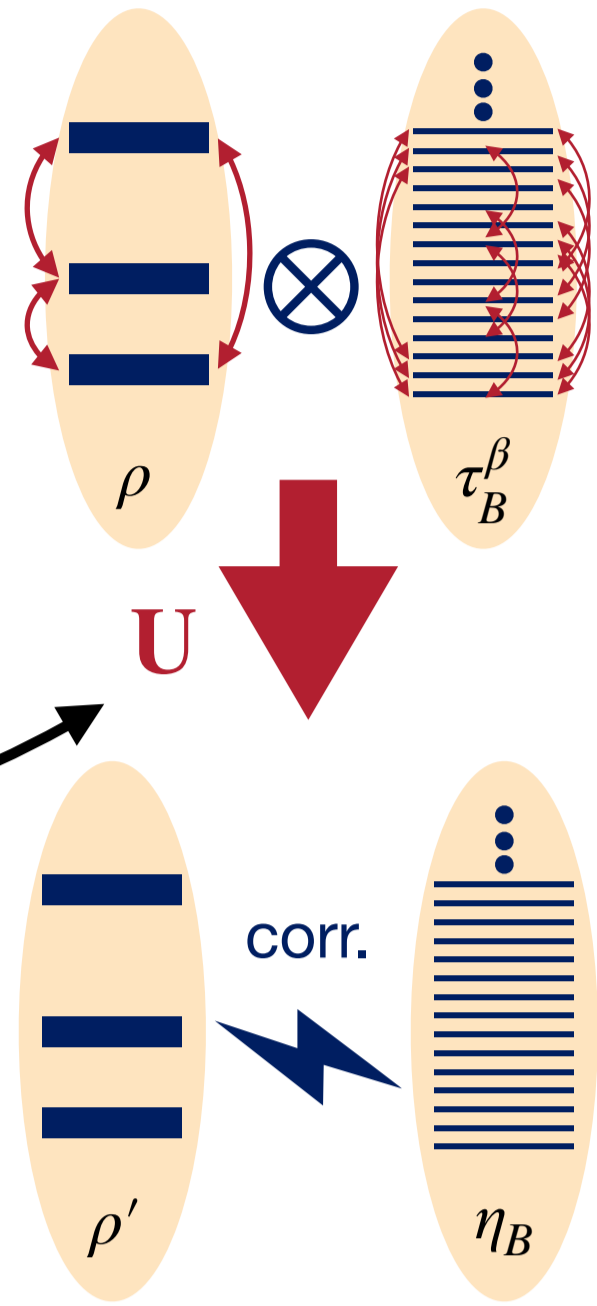
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Thermal operations (TO)

- any unitary U s.t. $[H_S + H_B, U] = 0$
- Gibbs states τ_B^β with any H_B are allowed

find ρ' given ρ ⇐ **easy!**

$$\rho \xrightarrow{\text{TO}} \rho' = \text{Tr}_B[U(\rho \otimes \tau_B^\beta)U^\dagger]$$

Given ρ, ρ', implement TO ⇐ **difficult!**
- finding U is nontrivial
- U requires control over many levels
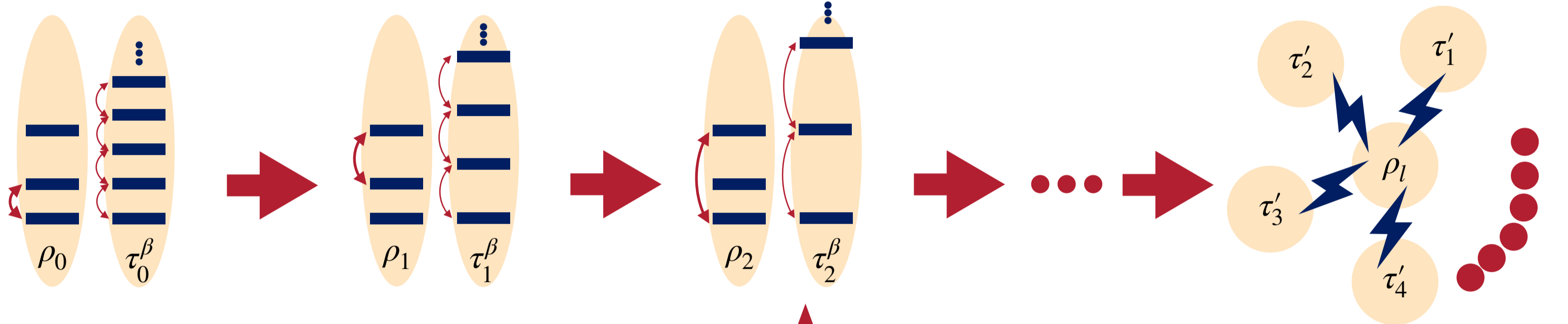
ρ ⊗ τ_B^β **U** ρ' corr. η_B

Elementary thermal operations (ETO)

Sequences of **2-lvl swaps** & their convex combinations
$\rho_0 \to \rho_1 \to \rho_2 \to \cdots \to \rho_l$

- each swap implementable w/ Jaynes-Cummings type interactions
- evolution naturally decomposable ❖might not always easy to find

ρ_0 τ_0^β ρ_1 τ_1^β ρ_2 τ_2^β $\cdots$ τ_2' τ_1' ρ_l τ_3' τ_4'

Exact catalysis
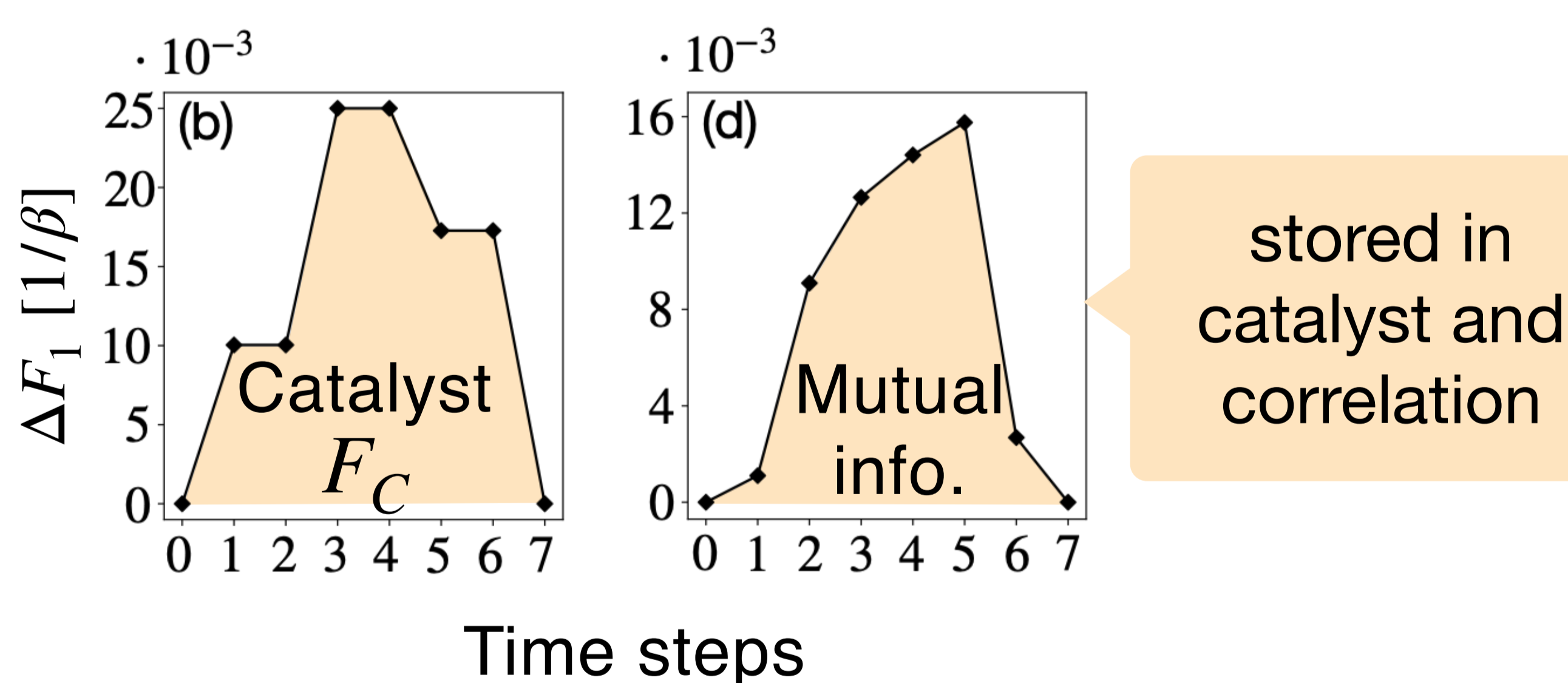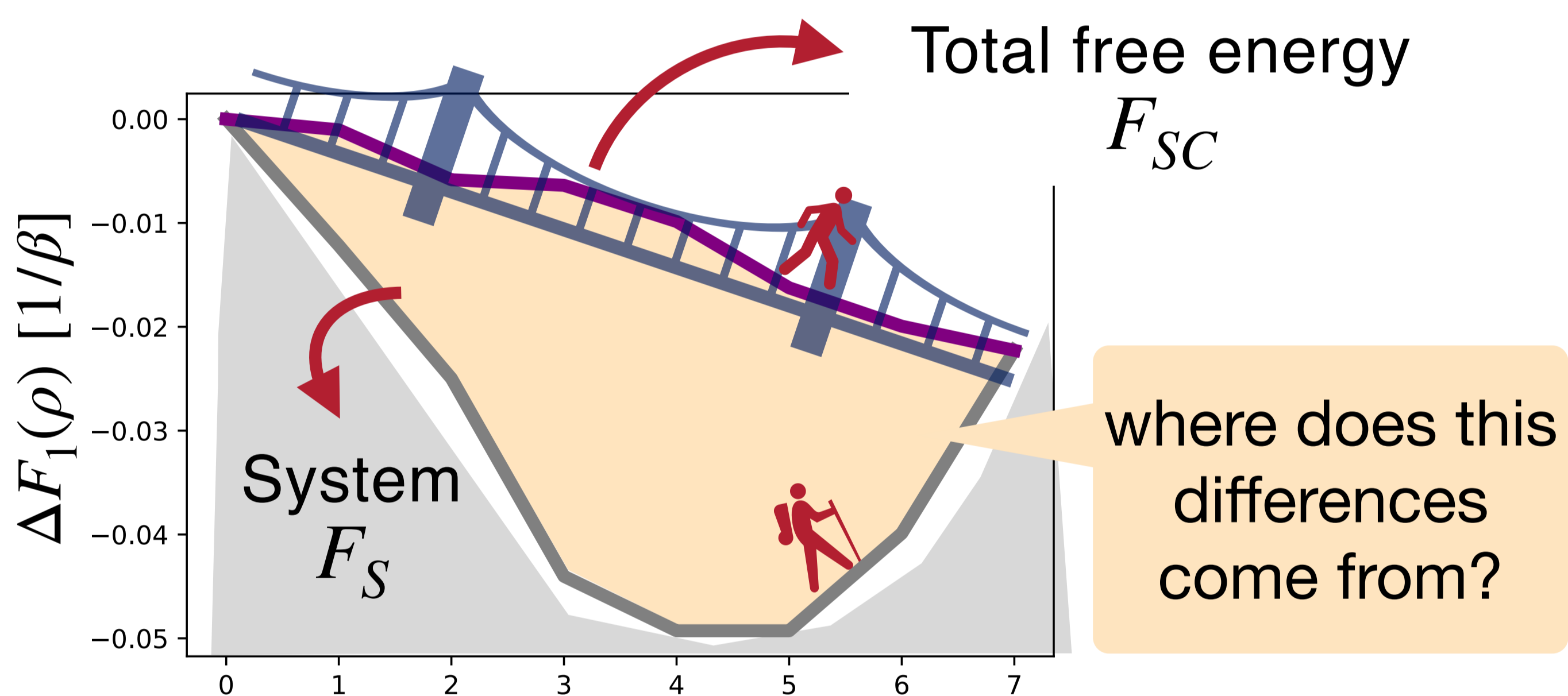
If $\rho_S \nrightarrow \rho_S'$ but $\rho_S \otimes \sigma_C \to \rho_S' \otimes \sigma_C$, σ_C: reusable ~ **catalyst**

'Exact' means: i) no error in final σ_C, ii) no correlation with ρ_S'

ETO ⊊ TO ⇐ ETO **strictly weaker** than TO
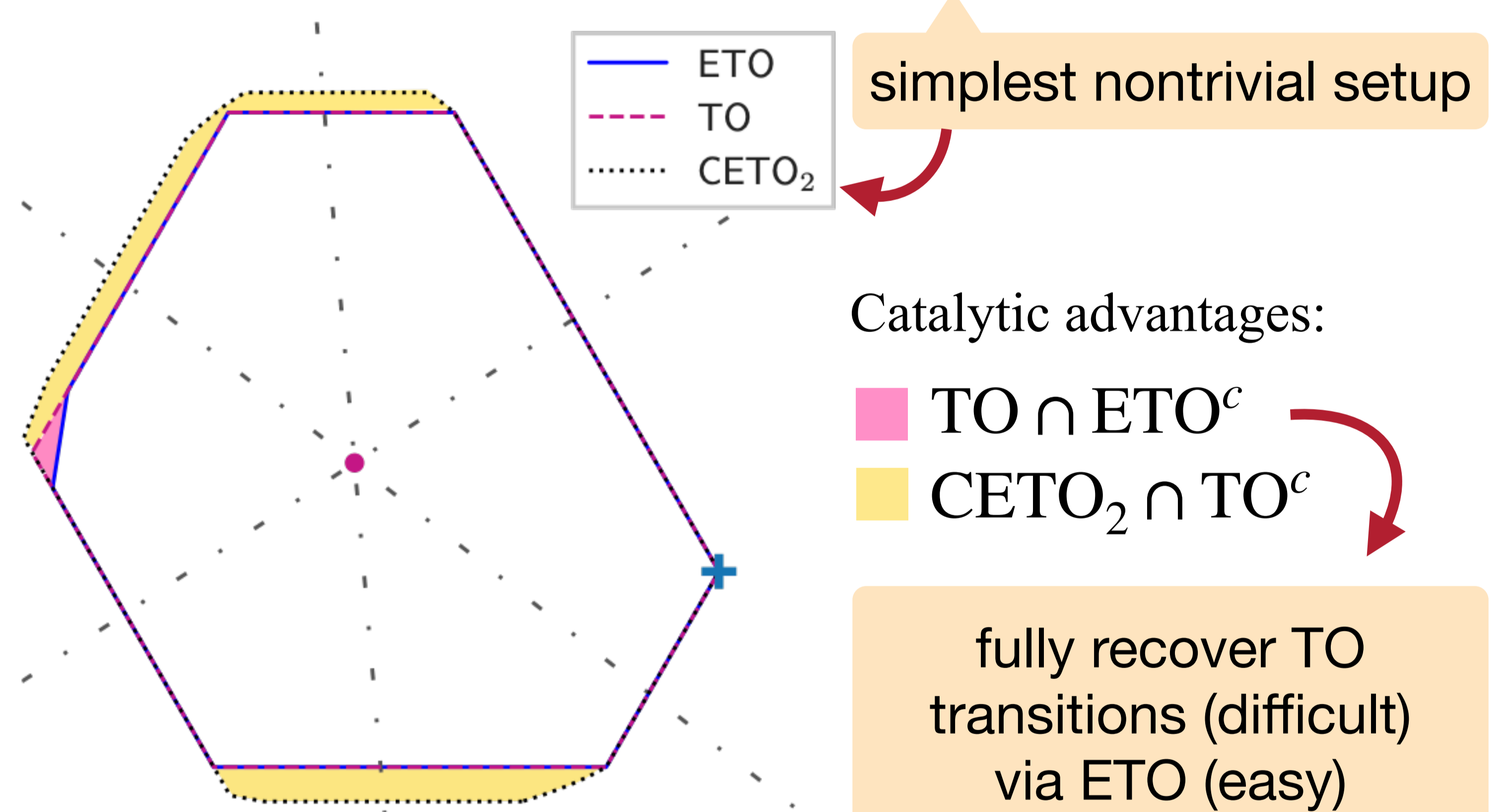
What if **catalysts** are allowed?

What is happening?

Total free energy F_{SC}

$\Delta F_1(\rho)$ [$1/\beta$]

System F_S

where does this differences come from?

ΔF_1 [$1/\beta$]

(b) Catalyst F_C

(d) Mutual info.

stored in catalyst and correlation

Time steps

❖ Generalized free energies **cannot increase** after a swap
➡ **system-reduced** F_S path is impossible w/o catalysts
➡ **total free energy** F_{SC} always decreases after each swap
➡ **catalyst-reduced** F_C change & **mutual info.** should be 0 at the end (exact catalysis)
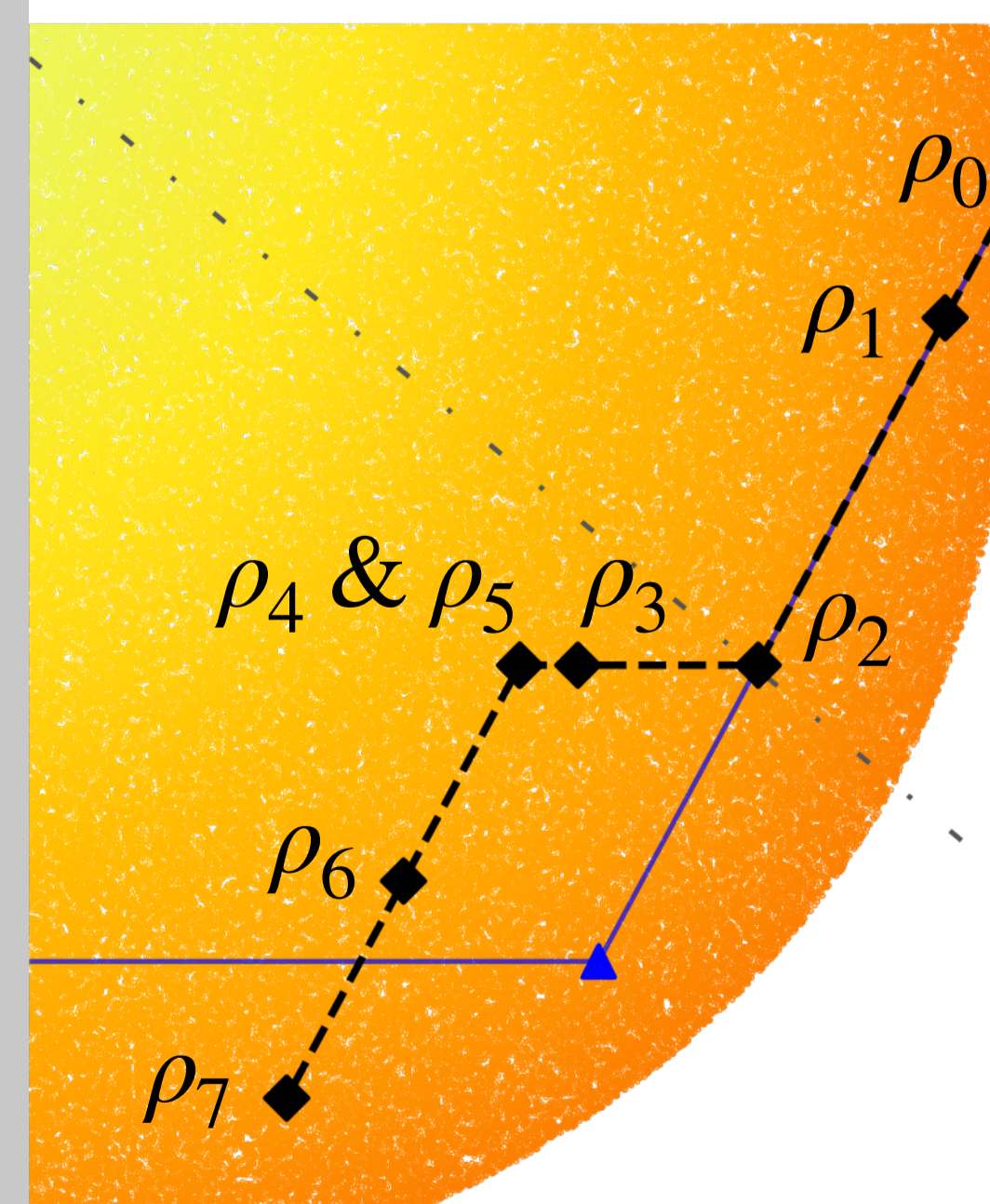
catalysts function as **"free energy storage"**

ETO vs. TO vs. CETO$_2$

∃catalytic advantage even when $d_S = 3$, $d_C = 2$

— ETO
-- TO
⋯ CETO$_2$

simplest nontrivial setup

Catalytic advantages:

■ TO ∩ ETO^c
■ CETO$_2$ ∩ TO^c

fully recover TO transitions (difficult) via ETO (easy)

Decomposing catalytic evolution: an example

▲ ETO ◆ CETO$_2$

ρ_0 ρ_1 ρ_4 & ρ_5 ρ_3 ρ_2 ρ_6 ρ_7

System levels: $E_k < E_l < E_m$
Catalyst levels: $E_1 = E_2$
Composite levels:
$E_{k_1} = E_{k_2} < E_{l_1} = E_{l_2} < E_{m_1} = E_{m_2}$
$\beta^{(x,y)}$: full swap, $M^{(x,y)}$: partial swap

$$\rho_0 \xrightarrow{\beta^{(k_1,m_2)}} \rho_1 \xrightarrow{\beta^{(k_2,m_2)}} \rho_2 \xrightarrow{\beta^{(l_1,m_2)}} \rho_3 \xrightarrow{M^{(l_2,m_2)}} \rho_4$$
$$\xrightarrow{M^{(l_1,l_2)}} \rho_5 \xrightarrow{\beta^{(k_1,m_1)}} \rho_6 \xrightarrow{\beta^{(k_2,m_1)}} \rho_7$$

Bonus: improvement of ETO computability

Finding ρ' reachable via ETO from ρ ⇐ **difficult!** (search over length-$l_{\max}$ swap sequences)
- improved $l_{\max} = d_S! \Rightarrow l_{\max} = \lfloor d_S!/(d_S - 2) \rfloor$ for d_S-dimensional system
- fully characterized $d_S = 3$ case (no need to search over sequences)

References

- M. Lostaglio, A. Alhambra, and C. Perry, Quantum **2**, 52 (2018)
- J. Son and N. Ng (In preparation)

And many more, including criteria for good catalysts, easily computable special cases, continuous-time F, etc.